“Oscillatory dynamics of city-size distributions in world historical systems” [1]

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for:

G. Modelski, T. Devezas and W. Thompson, eds. **Globalization as Evolutionary Process: Modeling, Simulating, and Forecasting Global Change**

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[1] Our special thanks to Constantino Tsallis, the inspiration for this study, who patiently taught Doug and Nataša the fundamentals of *q*-exponential concepts and methods and then answered questions as they proceeded through the substantive analysis, and to Peter Turchin for generously providing data and suggestions for analysis. Any errors however remain our own. Thanks also to Robert McC. Adams, George Modelski and William Thompson for critical commentary and suggestions. Support from EU project ISCOM, “Information Society as a Complex System,” headed by David Lane, Sander van der Leeuw, Geoff West, and Denise Pumain, is acknowledged for the city sizes project. We thank the leaders and member of the project for critical commentary. The larger project, Civilizations as Dynamic Networks, forms part of a Santa Fe Institute Working group for which SFI support is acknowledged. An early draft of the paper was presented to the Seminar on “Globalization as Evolutionary Process: Modeling, Simulating, and Forecasting Global Change,” sponsored by the Calouste Gulbenkian Foundation, meeting at the International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria, April 6-8, 2006. Sponsored by the Calouste Gulbenkian Foundation, meeting at the International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria, April 6-8, 2006. The reliability of the final analysis would not have been possible without the help of Cosma Shalizi, who derived and then programmed an MLE solution to the problems of estimating the parameters of historical city-size distributions, many of which had relatively small lists of largest city sizes once we got to the regional level.

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Abstract. Oscillatory patterns of expansion/contraction have long characterized the dynamics of demographic, economic, and political processes of human societies, including those of exchange economies and globalization. Major perturbations in city-size distributions are shown to exist for major regions in Eurasia in the last millennium and to exhibit some of the characteristics of cyclical oscillations on the scale of 100s of years as well as longer fluctuations, up to 800 years, between periods of collapse. Variations in timing, irregularities in amplitudes, and ups and downs in our measure appear to correlate with some of the peaks and troughs in urban population growth and show long-cycle correlations with J.S. Lee's (1931) sociopolitical instability (SPI) data on the durations of internecine wars for China.

We hypothesize an embedding of dynamical processes that runs from trading zone network sizes and city-size distributions that cycle roughly 200, 400, or 800 years, partly dependent on the severity of the decline. Our interpretation of city-size distribution oscillations is that they follow, with a generational time lag, rises and falls in the expansion/contraction of multi-connected trade network macro zones. More Zipfian city-size hierarchies tend to rise with trade network expansions and fall with contractions. These might be longer oscillations than Turchin's population ("secular") SPI cycles that average about 220 years. Turchin cycles seem to embed two leading polity cycles (Modelski and Thompson 1996) that average about 110 years. These are averages, and actual timings vary, but we give explanations for why such these average cycle-lengths might tend to diminish by half as each embedded process tends to operate at successively smaller spatial scales. We do find evidence that rise and fall in Silk Road connectivities between China and Europe had time lagged effects on the growth of power law tails in European urban hierarchies.

Time-lagged synchronies in the dating of phases for city distributions in different regions that are connected by multiple routes of trade, as noted tentatively by Chase-Dunn and Manning (2002:21), at least in the rising and more Zipfian phase, lend credibility to the existence of city-system rise and fall cycling. We focus here on central civilization, including China and Europe, and the Mid-Asian region between, and the world cities database. These data are likely to reflect changes in the macro regions connected by trade networks, where we would expect synchronization. Our three larger regions show dominant time-lagged effects of city-system rise and fall for Mid-Asia affecting China in this millennium, and China affecting Europe, at different time lags.

INTRODUCTION

Globalization, world-system, and historical dynamic theory offer complementary perspectives for the study of city systems as the politicoeconomic engine of interstate networks. Here we combine these perspectives to examine a dynamical perspective on systems of cities. Globalization theory applied to Eurasia in the last millennium (e.g., Modelski and Thompson 1996) focuses on centers of economic innovation and political power and their successive periods of rise and fall in dominance. Units of larger scale, as for example polities, are shown to operate at successively longer time scales in rise and fall than the economic innovation centers within those polities. World-system theory for similar regions and processes (e.g., Chase-Dunn and Hall 1997) differs in focusing as well on innovation at the peripheries of states and empires, that is, on the marcher or boundary polities that resist the encroachment of expanding empires. Marcher states that amalgamate to defeat the spread of empire often defeat polities formally organized on a much larger scale through superior cohesive or decentralized organization able to forge a superior combative skill or technology. World-systems theory often limits itself to the more prominent types of relations, such as trade in bulk goods and interstate conflict, that form distinct macroregional networks. The structural demographic approach to the political economics of agrarian empires (e.g., Turchin 2003, 2006) is capable of yielding a more dynamical historical account of how central polities rise and fall as their internal cohesion disintegrates with population growth into factional conflict and of how once dominant polities and economies contend with marcher states that coalesce into formidable opponents on their frontiers.

Several of the problems in extending these kinds of complementary approaches to globalization, world-systems, and historical dynamics relate to how networks – social, political, and economic – fit into the processes of change and dynamical patterns that are observed historically. One aspect of major problems that might engage network research involves how changing network fluctuations of long distance trade influence inter- and intra-regional dynamics. The Silk Road trade so important in the connections through the marcher states and later empires of Mongol Central Asia between China and the Middle East, for example, also facilitated the diffusion of economic inventions from East to West that were crucial in the rise of the European city system. These included paper money, institutions of credit, and vast new knowledge, weaponry, and technologies. Spufford (2002), for example, shows how important were the transmissions of innovations from China from a European perspective, while Temple (1987) summarizes the work of Needham (1954-2004) to show the debt of the West to China. A related problem, among many other open network problems in historical research is how regional and long-distance trade networks are coupled, along with conflicts and wars, to the rise and fall of cities and city systems. That is the problem we take up here.

We approach the problems of the rise and fall of commercial trade networks, regional city systems, regional conflicts, and the historical dynamics of globalization and world-system interactions in Eurasia, during the last millennium, with a concern for valid comparative measurement of large scale phenomena. Tertius Chandler (1987) and other students of historical city sizes (Pasciuti 2006, Modelski 2003, Bairoch 1958, Braudel 1992, and others) have made it possible to compare the shapes of city-size distribution curves. These are the data we examine here in order to gauge and compare the dynamics of city system rise and fall, both within distinct regions and as changes in one region (such as China) affect changes in others (such as Europe, to give but one example).

Our approach here is to divide up Chandler's (1987) Eurasian largest city-sizes data into three large regions – China, Europe, and the Mid-Asian region in between – and measure variations over time that depart from the Zipfian rank-size distribution. Zipfian rank-size is the tendency for cities ranked 1 to n in size to approximate a size of M/r where r is a city's rank compared to the largest city and M is a maximum city size M that gives the best fit for the entire distribution (this formulation allows the rank 1 largest city size S_1 to differ from its expected value under a Zipfian fitted to an extensive set of the larger cities). The Zipfian distribution has been taken to be a recurrent and possibly universal pattern for city sizes as well as many other complex system phenomena. What we find for Eurasia and regions within Eurasia is that there are systematic historical periods that show significant deviations from the Zipfian. Some of these deviations show the characteristics of a regional collapse of city systems from which there is eventual recovery (unlike cataclysmic collapse exemplified by the Mayan cities system).

The periods of rise and fall of city systems for each Eurasian region, however, are different. This allows us to test the hypothesis that the rise and fall measure for China anticipates with a time lag the rise and fall measure for Europe, which is a prediction for the period starting in 900 CE consistent with Modelski and Thompson (1996), Temple (1987) and Needham (1954-2004). Finally, for the region of China we have sufficient time-series data to test the predictions from the historical dynamics model of Turchin (2005). This allows some limited results on whether some of the same processes are operative for the rise and fall of city systems as for the historical dynamics of state and empire rise and fall.

In Part I we pose the problem of instabilities in city sizes and systems drawing on Chandler's data for 26 historical periods from 900 CE to 1970. Part II examines ways of measuring departure from Zipfian distributions of city sizes and introduces the data used for city sizes and possible correlates of city system change. Part III gives the results of the scaling of city sizes for different regions so as to measure city system changes. In Part IV we examine the time-lagged interregional cross-correlations for these measures, and summarize results for crossregion synchrony. Part V examines correlations and time-lags between our three Eurasian regions and for other variables related to known historical oscillations where we have adequate data for hypothesis tests. The variables tested include such variables as trade connectivity, internecine warfare within China and development of credit and currency systems that facilitate international exchange as well as innovative national markets. VI concludes with a summary and implications of the findings.

PART I: CITY SYSTEM INSTABILITIES p. 3

Jen (2005:8-9) defines *stability* in terms of dynamical recoveries from small perturbations that return to an original state. Seriousness of the question of city system instability and of major departures from the Zipfian derive from the assumption that city economies are organized as networks that involve trade and war, and depend on innovation to join the leading economic or political sectors of more global networks. The two main factors that make for instability are competition and population growth.

Economic competition, aided by power politics, tends to make for oscillations that may return to what might be called structural stability. That is, they make for economic and political limit cycles rather than conservative stationarity. Populations of polities, empires, regions, and global world systems, also exhibit limit cycles if we average out trends of population growth, e.g., over the last several millennia. Jen defines *structural stability* as the ability to return from instability through other dynamics than the original (e.g., by varying external parameters) that are qualitatively similar to the original dynamic, as for example the Lotka-Volterra type of oscillatory limit cycle. While economic and political systems are not stable in the strict sense they may have the resilience to return to structural stabilities as they pass through oscillatory limit cycles with differing but qualitatively similar dynamics. Major population growth trends, however, as they interact with dynamical oscillations or limit cycles, may lead to *structural instability*, an inability to return to stability even through other dynamics than the original but that recover qualitative similarity to the original. The imperative of incessant competitive innovation for successful cities and city systems forms part of what leads to overgrowth of population relative to resources and to subsequent system crashes. Historically, these instabilities lead eventually to industrial revolutions that, rather than conserve materials and energies, may push extravagant degradation of resources into dynamically irreversible crises such as global warming and problems of structural instabilities. Unless innovation turns toward conservation the problems created will not be solved in the next century or possibly not in next millennium. The issues here are ones of scale, expansions of scale (size of cities, size of polities and empires, size of economies), the dynamic interactions that operate at different scales, and how these couple spatially and temporally (as described, for example, in Modelski and Thompson 1996).

The first questions of this study, then, are whether city systems as central economic actors and sites for multitudes of agents are stable or unstable, and if unstable, what kinds of models are appropriate for consistency with their dynamics. The thesis here is that it is not just individual cities that grow and decline but entire regional (and global) city systems. Here, drawing on our earlier work (White, Kejžar, Tsallis and Rozenblat 2005), Michael Batty (2006:592) states our case for us. "It is now clear that the evident macro-stability in such distributions" as urban rank-size or Zipfian hierarchies at different times "can mask a volatile and often turbulent micro-dynamics, in which objects can change their position or rank-order rapidly while their aggregate distribution appears quite stable…." Further, "Our results destroy any notion that rank-size scaling is universal… [they] show cities and civilizations rising and falling in size at many times and on many scales." What Batty shows, using the same data as do we for historical cities (Chandler 1987), is legions of cities in the top echelons of city rank being swept away as they are replaced by competitors, largely from other regions.[5]

PART II: DATA p. 4

City Size Data for Historical Eurasia

Chandler's (1987) database on historical city sizes is complemented by overlapping UN population data from 1950 to the present (in the interest of brevity we do not present these results here). Chandler reconstructed urban populations from many data sources. These included areas within city walls times number per unit area (see Appendix A), connected house-to-house suburbs lying outside the municipal area, data from city histories provided by city librarians, estimates from numbers of houses times numbers per house, and the cross-checking of different estimates (see Pasciuti and Chase-Dunn 2002). From 900 CE to 1970 his size estimates cover

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[5] Noting from the shared database that the top echelon of cities in a single region may be swept away in a short period by interregional competition, Batty refers to our work on instabilities at the level of city systems.

over 26 historical periods, usually spaced at 50 year intervals, always comprise a set of largest cities suitable for scaling in a single period. These data include 80 Chinese, 91 European, and in between a much larger number of Mid-Asian cities.

Figure 1 shows numbers of cities in the dataset for in each period when they fall below 21. As numbers decline from 1200-1650 for China, for example, China becomes less hegemonic as many of the top 75 world cities appear in other regions of the world. Resurgence of China begins to occur after the end of the last (Qing) dynasty.

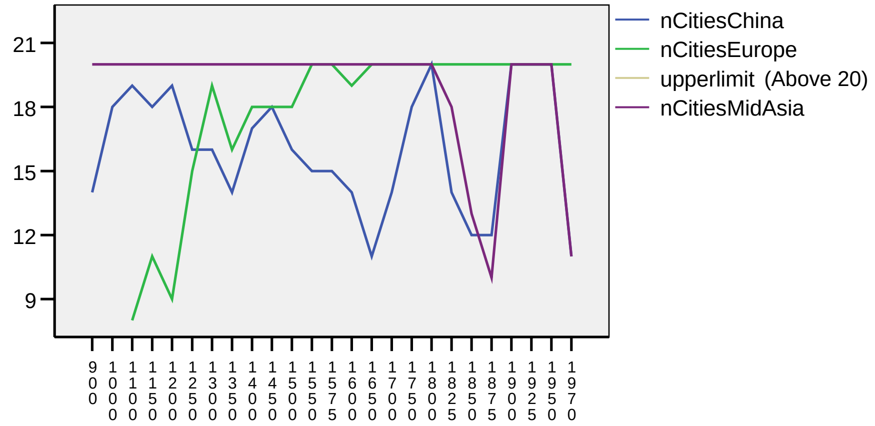

Figure 1: Number of Cities in Top 75 World Cities in each region when they fall below 21

Trade Routes (Eurasia). The total length of the Eurasian long-distance trade routes between 3500 BCE and 1500 CE at 50 year intervals have been calculated by Ciolek (2005) from trade-route maps drawn by Sherratt (2003). World-system pulsations in expansion and contraction of trade routes are shown by Turchin (2007). Turchin calculated a connectivity index for the Silk Routes between China and England. His data also show how instances of epidemics are concentrated near the high points of trade before periods of collapse.

Sociopolitical Instability: Internecine Wars (China only)

Turchin (2003:164) transcribed J. S. Lee's (1931) coded 5-year interval data on internecine wars in China from 221 BCE (unification by the first Ch'in Dynasty Emperor) to 1929 to create a ten-year sequential intensity index to 1710. D.R. White converted Turchin's codes into a 25-year index running from CE to 1700 and coded 1725-1925 directly from Lee (1931). Lee coded the period to the end of the Ming Dynasty from a remarkably systematic inventory of conflicts by Chih Shao-nan from the Tih Wang Nien Piao and checked for accuracy by Tung-Kien (Lee

1931:114). The Tabula Annua (Seikainenkan), cross-checked and supplemented by the Tai Ping Tien Kuo Chan Ssi proved a reliable record of Ch'ing Dynasty wars, with those following from Lee's memory. The systematic pattern discussed by Lee for his graphs is one of two 800-year periods (200 BCE-600CE, then to 1385 CE) in which many more of the intra-territorial China conflicts occur in the last 400-500 than in the early 400-300 years, and a partial repetition up to 1927 of that same pattern. The other evident pattern is for conflicts to become much more likely at the transitions between dynastic periods.

Total Population and Population Change Data (Eurasia)

The early data on Chinese population from 900 to 1300 are controversial. White had data from Chao and Hsieh (1988), provided by Turchin (2003:164-165), he consulted Ho (1956, 1959), Steurmer (1980), Mi Hong (1992), Durand (1960), Heilig (1997, 1999, 2002), and the radical revision proposed by Heijdra (1995) and Mote (1999) that was critiqued by Marks (2002), and others. Given the uncertainty in absolute figures, we coded a binary variable for each 25-year period where 1 is given for a date at which there is a population peak before collapse, with 0 otherwise. The different total population estimates available to us for China over our full time frame agreed very closely as to where these population peaks occurred. In some cases two adjacent peaks were indicated.

Turchin (2006, 2007) provides population, carrying capacity, detrended population, and a misery index (inverse wages) that is useful for England into comparison to our European city data.

Monetary Liquidity (China only)

We devised a robust index of monetization (liquidity) for China from Temple (1986:117-119,168), coding data on credit, paper money, and banking for 900-1700 into a scale from 0-10, treating inflation as lower liquidity.

PART III: THE Q/β SCALING AND HYPOTHESES p. 6

Measuring Departures from Zipf's Law for City Size Distributions

We begin by examining the city size distributions of macroregions in the Eurasian continent, over roughly the last millennium, which is the millennium of modern globalization. We examine the extent of instability of city systems in ways that are visually evident by inspection of changes of the shapes of Eurasian city size distributions, and measured by a shape index. Figure 2 shows a semilog graph of the cumulative rank-size distribution for divisions of most of Eurasia (excluding Japan/Korea) into three regions: China (c900-c1970), Europe (e900-1970) and the Mid-Asian (m900-c1970) remainder. The curve to which power-law distributions should correspond is shown by the top (ZipfCum) power-law curve. Cumulative population size is logged on the y axis and the x axis is city size rank. The Zipfian curve forms a straight line when rank is also logged, with a Pareto log-log slope of 2. As can be seen visually, there is some departure from perfect parallelism in the empirical curves: some lines are more curved or less curved for the top cities than the Zipfian, most lines are flatter that the Zipfian for the smaller cities and many of the curves bend at different city ranks.

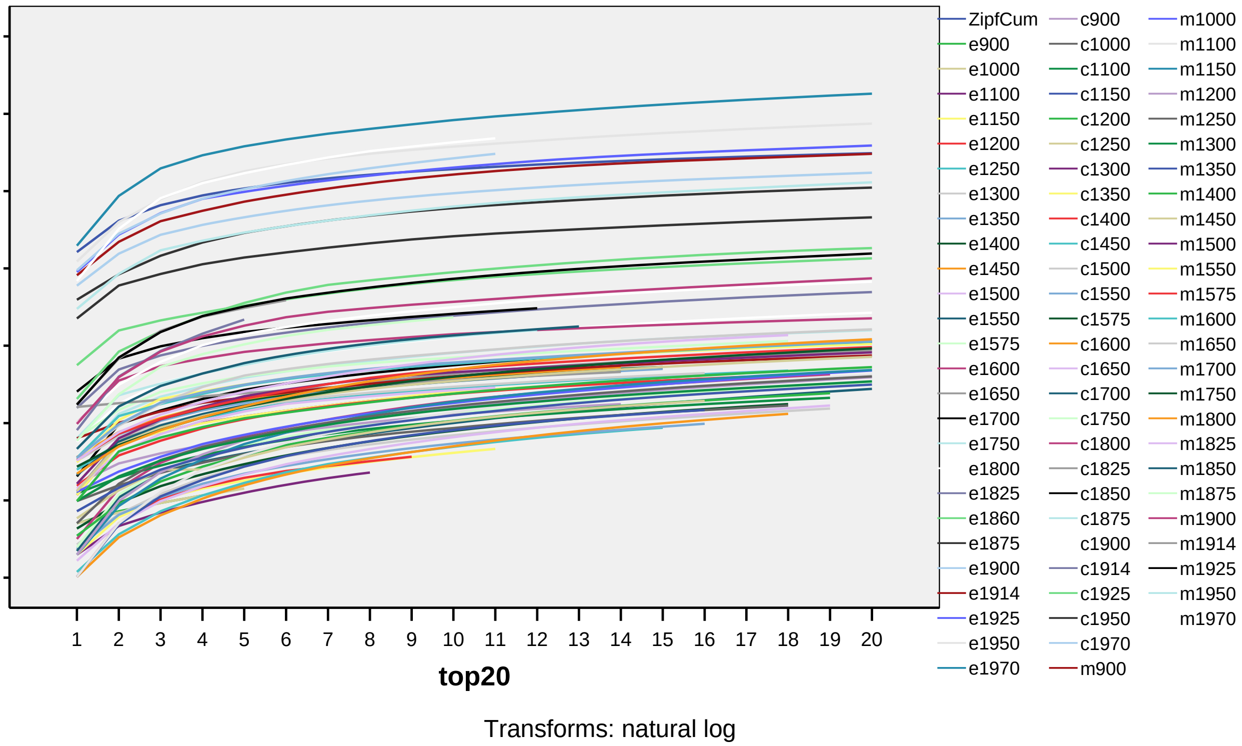

Figure 2: The Chandler Rank-Size City Data (semilog) for Eurasia (Europe, China, Mid-Asia)

Figure 3 shows the same data (all three regions) in a log-log plot where power-law city distributions would all be straight lines and Zipfian distributions would all have the same slope. The lines are neither parallel nor of the same slope, nor do they have curvatures in the same places. Our measurements of the properties of these distributions will be aimed at the hypothesis that these variations provide indicators useful to showing how city system fluctuations fit into economic and historical dynamics.

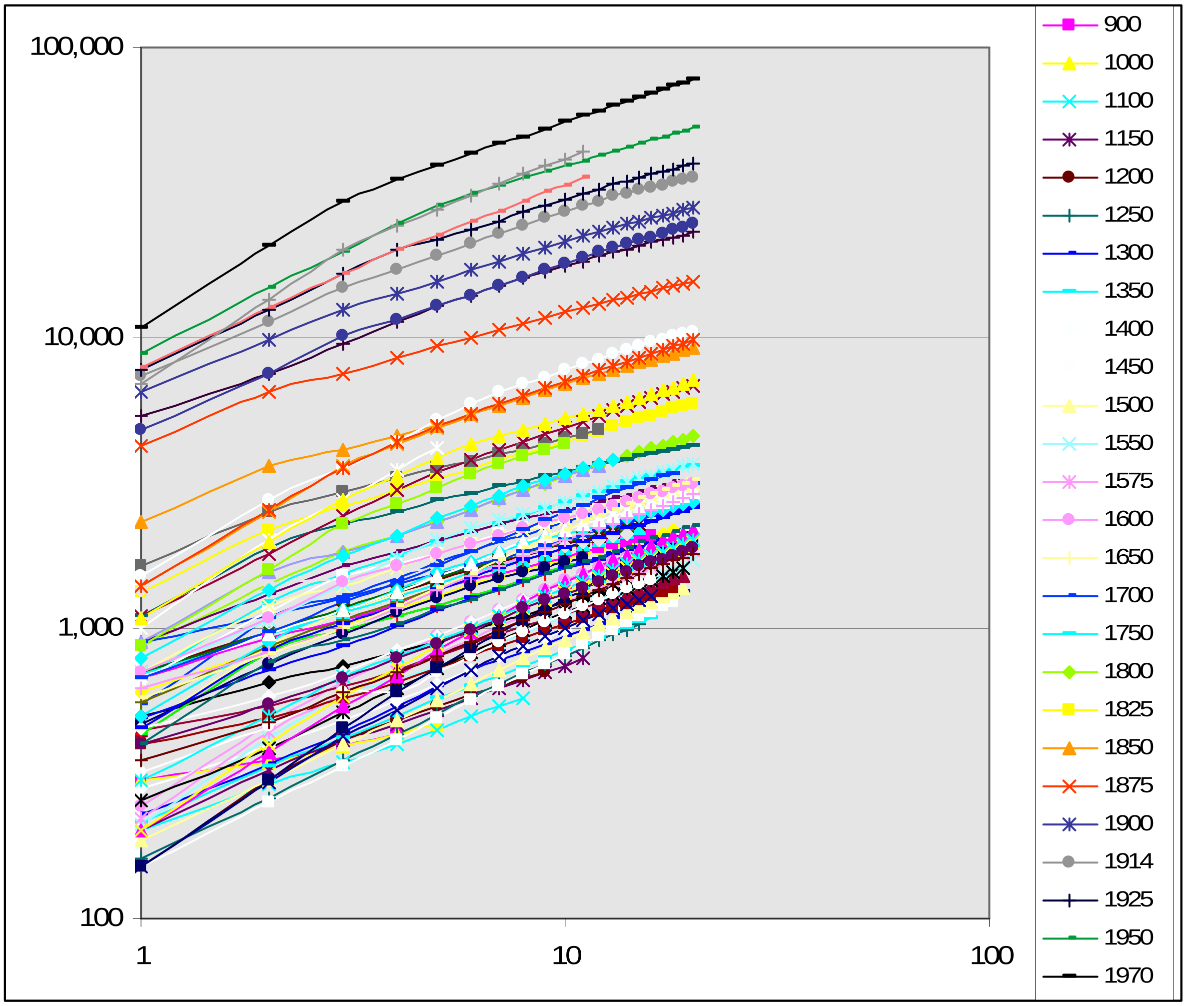

Figure 3: The Chandler Rank-Size City Data (log-log) for Eurasia (Europe, China, Mid-Asia)

We use two measures of how these curves vary. Each measurement model is based on a standard continuous function with parameter fitting based on recasting the data as a cumulative probability in its complementary form. $P(X \geq x)$ is the probability that an urbanite will reside in a city of at least size x and each model, as a cumulative complementary distribution function (CCDF), is aimed at capturing the shape of the empirical $P(X \geq x)$. It is especially important to use MLE or maximal likelihood estimates in fitting the parameters of distributional models from small samples. MLE estimates are unbiased, meaning that with *n* independent samples of the same data, the expected values of the estimated parameters for each sample would converge to the true parameter values.

The first measure is the standard (type I) Pareto distribution with a single parameter β where the Zipfian is the special case of $\beta = 2$:

$$P_{\beta}(X \geq x) = x^{-\beta} \qquad (1)$$

Fit to this distribution captures the extent to which the lines are straight in Figure 3.

Fit to a second function, a type II generalized Pareto distribution with a cut-off σ (Arnold 1983), captures the extent to which a given log-log distribution is curved. This function allows recognition that Zipf's and power laws for city sizes almost always have one or more cut-offs of a lower size below which the power-law changes or ceases to apply.[6] This function fits to the urban distribution a shape parameter Θ, analogous to β, and a scale parameter σ.

$$P_{\Theta,\sigma}(X \geq x) = (1 + x/\sigma)^{-\Theta} \qquad (2)$$

We were fortunate to have Cosma Shalizi (2007) write an R program for our use of maximal likelihood estimation (MLE) methods (Arnold 1983) for estimating parameters Θ,σ in equation (2).[7] He also provided us with an R program for standard MLE with the Pareto distribution. The type II Pareto has an extra σ parameter, but given unbiased ML estimation of the Θ,σ parameters, there are advantages of the extra cut-off parameter σ that specifies where the curve breaks for a power-law or Zipfian tail:[8]

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[6] For example, excluding two primate city outliers, the next largest 16 cities for 1998 in the U.S. (over 11 million) show a steep log-log slope, those ranking down to .5 million show a shallower slope, those to 1 million a much shallower slope, and then the power-law disappears altogether (Malacarne et al. 2001:2).

[7] Commentary on our White et al. (2004) paper on the present topic elicited the suggestion from Cosma Shalizi that we should consider MLE estimates for fitting q-exponentials to city size data. Given our small sample sizes from the Chandler data as we moved from a world sample (75 data points) to regional samples, we called on Shalizi in late 2006 to derive the MLE equation for us, which he did,. He then found earlier derivations such as those of Arnold (1983) and others. We are greatly indebted to Shalizi for writing functions in R to make MLE (unbiased) estimates of the q-exponential parameters and bootstrap estimates of the standard errors of these estimates.

[8] There are other advantages that we do not exploit here:

a) Y_q estimates an expected "largest city size" M consistent with the body of the size distribution. This requires simultaneous estimation of M and Y(0) to solve $Y(0)\, P_{\Theta,\sigma}(X \geq M) = M$.
b) The total urban population can be estimated from Y_q without having data on all smaller cities, although this feature is not utilized here.
c) Equation (1) and Y_q may be fitted *without* the largest city so as to derive an expected size for the largest city given our model.
d) This gives our model a ratio measure of the largest city size to its expected size from Y_q.
e) Y_q has a known derivative $Y_q'(x) = Y0/\kappa[1-(1-q)x/\kappa]^{(q/(1-q))}$ giving the slope of the curve $Y_q(x)$ for any city size x.
f) Solving for $Y_q(M) = M$ for the estimated largest city size M consistent with Y_q gives $Y_q'(M)$ as the slope of the Y_q at M and converges with $\beta = 1/(q-1)$ for the Pareto power-law slope.

a) It can fit an exponential function or a collapsed tail in cases where a power-law or Zipfian tail is inapplicable.

b) Equation (1) and its parameters Θ,σ are transformations in 1-to-1 functional correspondence with the q-exponential $Y_q(S \geq x) = Y0\ (1-(1-q)x/\kappa)^{1/(1-q)}$ model (see Tsallis 1988) and its shape (q) and scale (κ, or kappa) parameters. Here, $\sigma = \kappa/(q-1)$ and $\Theta=1/(q-1)$. The Y_q model has been previously used (Malacarne, Mendes, and Lenzi 2001) for city-size distribution scaling.

c) With MLE the size of the sample of largest cities does not bias the parameter estimates.

d) Bootstrap estimates of the standard error and confidence limits of the q,κ parameters are provided by Shalizi's (2007) R program for MLE. The standard errors are typically very small for q and the true values of the parameters are likely to be within these limits.

e) MLE runs can be batched for multiple datasets, and the MLE commands are easy to copy and paste into R, e.g.,

```
china.900 <- c(500,150,90,81,75,75,70,65,60,58,49,47,40,40)
china.900.tsal.fit <- tsal.fit(china.900,xmin=40) # Assigns the results of the fit to the object
china.900.tsal.fit # Displays the estimated parameters and information about the fit
(these estimates run in seconds)
china.900.tsal.errors <- tsal.bootstrap.errors(china.900.tsal.fit,reps=100)
china.900.tsal.errors # Displays the bootstrapped error estimates
(the latter estimates run in minutes)
```

HYPOTHESES

Several linked hypotheses build on one another, each supposing the previous hypotheses to be supported, and each adding greater specificity in relation to the parameters of the two models, β for the Pareto, and q, κ, and Y(0) for the q exponential or Type II Pareto:

H1. The q-exponential model provides appropriate measures to fit city size distributions: q for slope in the tail, σ for crossover to a more exponential body, and the total urban population asymptote, Y(0).

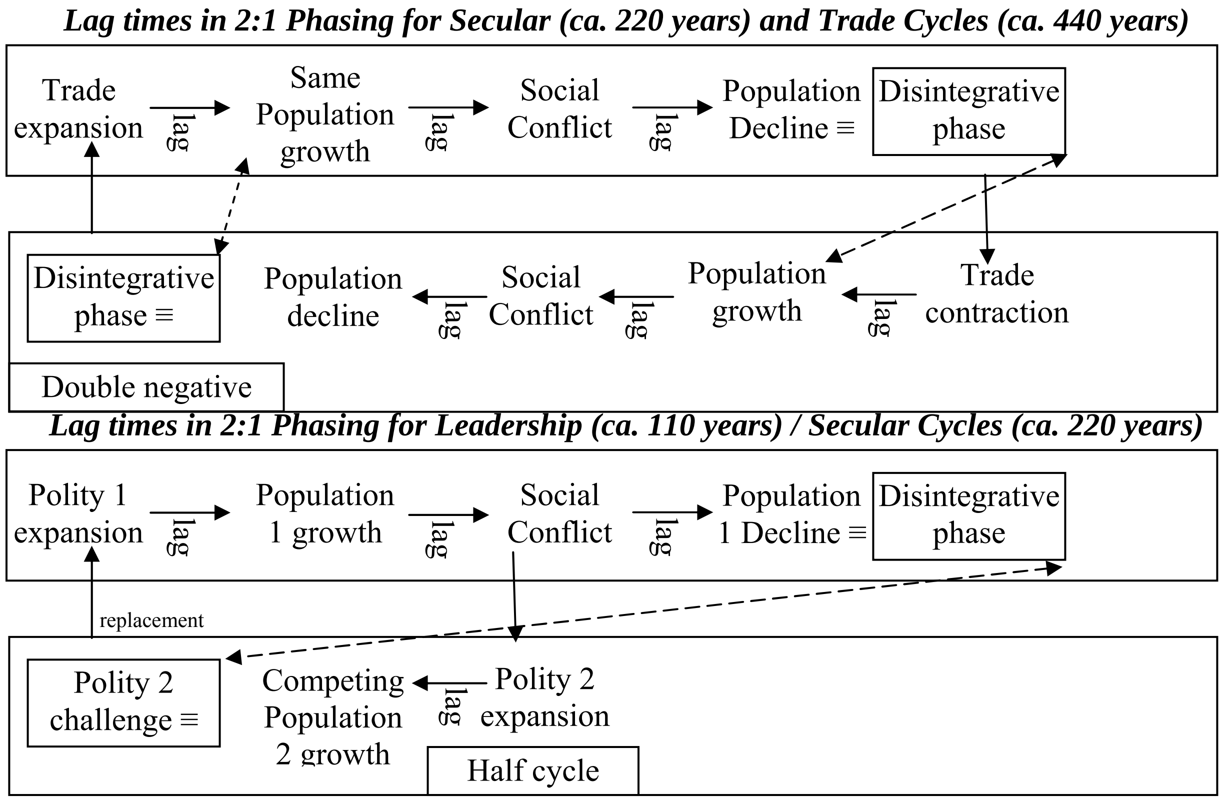
H2. Variations in q are deep indicators of rise and fall of urban systems, analogous to those of a β Pareto (power-law) coefficient for cities.

H3. Evaluated in the context of population peaks and troughs and SPI fluctuation, an optimal or normal state of q for city distributions will reflect a body of the distribution that is consistent with a Zipfian tail of $\beta=2=1/(q-1)$ where, solving for q, $q=1.5$. This should be the average value of q for the historical period.

H4. Variations in *q,* as a measure of rise and fall of urban systems, are strongly affected by population and sociopolitical instabilities in ways that reflect deviations up or down from an average *q* for a long historical period.

H5. Variations the length of oscillatory times for processes involving the successively larger spatial scale of local economies, innovative economic regions, and contention for leading polities (Modelski and Thompson 1996), the population/instability cycles of structural demographic secular cycles, and the cycles of empires (Turchin 2003, 2005, 2006) and global wars, and, finally, city systems (as studied here) are also successively longer in duration.

H6. These embedded or "stacked" spatiotemporal scales tend to occur in political economies according to a rough algorithm of doubling, diagrammed in Figure 4. The doubling is hypothesized to result from interaction between an internal endogenous dynamic and an external dynamic of pairing. In the top diagram below shows a possible explanation for a doubling of cycles between population rise and fall (smaller scale, faster cycle) and trade network rise and fall (larger scale, slower cycle. These couple through (a) an internal endogenous dynamic, the structural demography/secular cycle historical dynamic of Turchin (2003, 2005) where population rise and fall interact endogenously through scarcity and plentitude with sociopolitical instability (internal competition) and (b) a competitive dynamic between neighboring units in trading networks, where periods of scarcity in one region might be offset by economically advantaged neighbors with different sets of resources. Here the combination of a disintegrative secular phase and a contracted network regionally constrained by polity conflicts at the borders of region seems to be sufficient to weaken the border-polity conflicts, and clear the way for more globalized trade patterns to reemerge after being long suppressed in the second of the internal cycles. The second diagram repeats this logic for the endogenous dynamic of polities at a smaller scale and competing polities (populations) at a larger scale. Here within the shorter half-cycle in the secular oscillation (a shorter loop than the first diagram) is a time at which the leading polity is weakened in its growth cycle by social conflict, allowing a competing polity to expand and challenge the renewal of the first polity for leadership. A third embedded loop might involve the temporally variable polity expansion cycle in a population growth phase that might couple to leading economic sector growth for a leading polity. The key processes here are those that involve a weakness to emerge in the internal cycle that facilitates strengthening of the external cycle in a series of nested loops that draw on one another fractally by halvings and doublings.


Lag times in 2:1 Phasing for Secular (ca. 220 years) and Trade Cycles (ca. 440 years)

Lag times in 2:1 Phasing for Leadership (ca. 110 years) / Secular Cycles (ca. 220 years)

Figure 4: Internal/external completive dynamics exemplified at different spatiotemporal scales

SCALING RESULTS

Using visual and statistical evidence for changes in the shape parameter, White, Kejžar, Tsallis and Rozenblat (2005) were able in an earlier study to date six Q-periods in Eurasia over the last millennial period. These changes and periods were seen to be related to the framework for studying globalization developed by Modelski and Thompson (1996). In studying multiple regions we take a more detailed and dynamical view of this relationship.

Figure 6 shows the q and β slope parameters fitted by MLE for the regions of China, Europe and the region between (Mid-Asia, from the Middle East to India),. Here, a Zipfian tail would have q=1.5 and β=2. The horizontal line shows that this slope and shape is approximated more recently in the early modern and modern period. We also show a normalized minimum of q and β in which we divide q by 1.5 and β by 2.0 to normalize for the Zipfian.

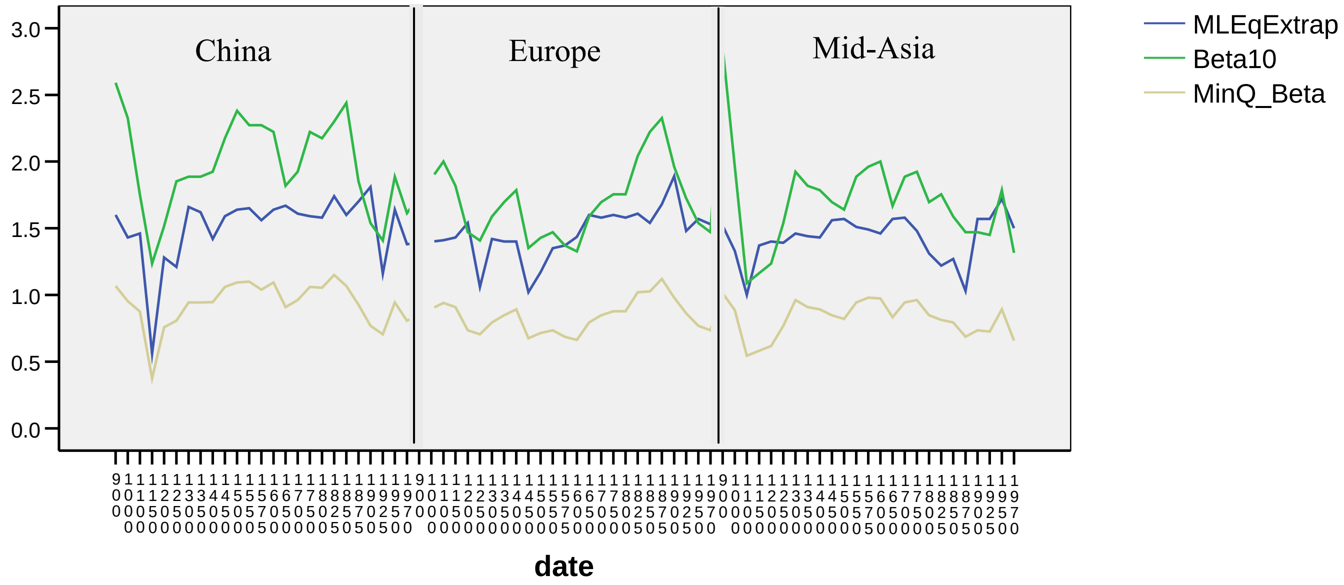


Figure 5: Values of *q, β*, and their normalized minimum

Table 1: Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Std.Dev/Mean
MLEqChinaExtrap	25	.56	1.81	1.5120	.25475	.16849
MLEEuropeExtrap	23	1.02	1.89	1.4637	.19358	.13225
MLEMidAsIndia	25	1.00	1.72	1.4300	.16763	.11722
BetaTop10China	23	1.23	2.59	1.9744	.35334	.17896
BetaTop10Eur	23	1.33	2.33	1.6971	.27679	.16310
BetaTop10MidAsia	25	1.09	2.86	1.7022	.35392	.20792
MinQ_BetaChina	25	.37	1.16	.9645	.16247	.16845
MinQ_BetaEurope	23	.68	1.26	.9049	.15178	.16773
MinQ_BetaMidAsia	25	.54	1.01	.8252	.13217	.16017
Valid N (listwise)	19					

Mean values for q in the three regions vary around q=1.5±.07, consistent with a Zipfian tail, and similarly for variations around β=2 ±.07 for the top 10 cities Pareto slope. Statistical runs tests of whether the variations around the means are random or patterned into larger temporal periods are shown in Table 2 and 3. The runs tests reject the null hypothesis (p<.01 for Europe, p<.05 for Mid-Asia, and p<.06 for China; p<.00003 overall).

Table 2: Runs Tests at medians across all three regions

	MLE-*q*	Beta10	Min(*q*/1.5, Beta/2)
Test Value(a)	1.51	1.79	.88
Cases < Test Value	35	36	35
Cases >= Test Value	36	37	38
Total Cases	71	73	73
Number of Runs	20	22	22
Z	-3.944	-3.653	-3.645
Asymp. Sig. (2-tailed)	.0001	.0003	.0003

Table 3: Runs Test for temporal variations of *q* in the three regions

	mle_Europe	mle_MidAsia	mle_China
Test Value(a)	1.43	1.45	1.59
Cases < Test Value	9	11	10
Cases >= Test Value	9	11	12
Total Cases	18	22	22
Number of Runs	4	7	7
Z	-2.673	-1.966	-1.943
Asymp. Sig. (2-tailed)	.008	.049	.052

a Median

The time periods of successive values above and below the medians represent the rise and fall of *q* to Zipfian or steeper-than-Zipfian slopes alternating with low-*q* periods with truncated tails of the distributions. Relatively long city-slump periods occur in the medieval period for all three regions, a second slump occurs in Europe in 1450-1500, another in Mid-Asia in 1800-1850, and one in China in 1925 (not shown) when to *q* falls to 1.02. As shown by the lower dotted line in Figure 6, values of *q* below 1.3 might be considered as approach a city system crash or destruction of primate cities. China and Europe experience an abnormal rise in *q* in 1900 beyond 1.7 (upper dotted line). This results in a thin tailed distribution (extreme primate cities) that might be considered as a different kind of city system crisis. Some crashes have to do with wars, like the Song loss of their capital to the Jin in 1127. Global wars noted by stars on the lines in Figure 6 might have to do with the punctuations of these periods, but we are unable to evaluate that question statistically. Crashes in *q* usually occur at long intervals (as in **bold**) , as in Figure 5, with *β* falling at shorter intervals. The dates below roughly correlate with secular cycles (Turchin 2007) and Modelski-Thompson (1996) globalization processes:

Mid-Asia:	**1100,**		1450		**1825-75, 1914**	**(major**/minor **urban crashes)**
China:	**1150-1250,**			1650	**1925**	**(major**/minor **urban crashes)**
Europe:		**1250,**	**1450-1500,**		1950?	**(major**/minor **crashes)**
Cycle	1	2	3	4	5	(Modelski-Thompson 1996: Table 8.3)
Break	950	1150	1430	1640	1850	

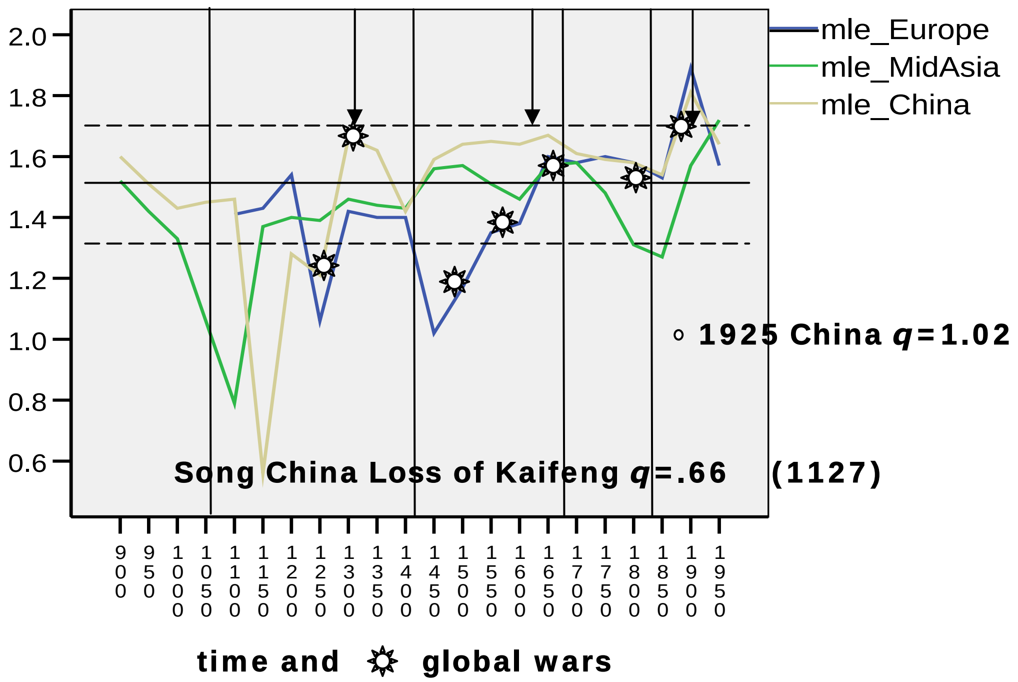
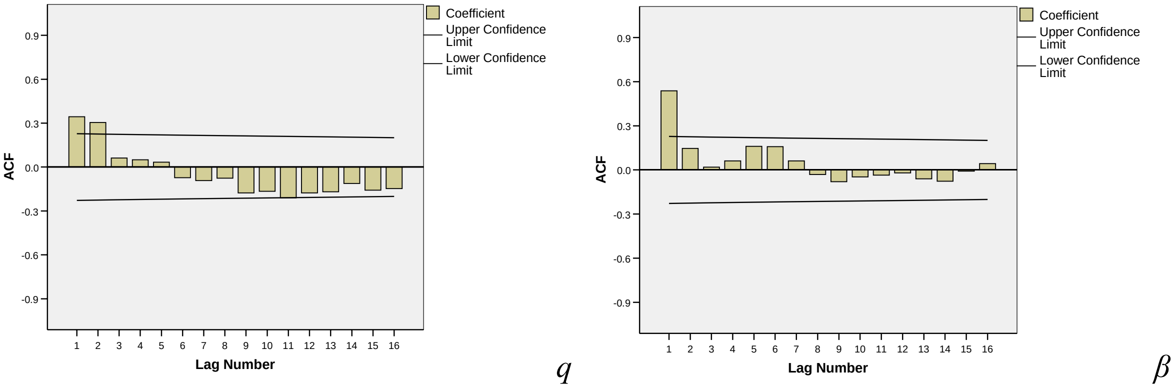


Figure 6: Fitted *q* and *β* parameters for Europe, Mid-Asia, China, 900-1970CE, 50 year lags. Vertical lines show approximate breaks between Turchin's secular cycles for China and Europe Downward arrow: Crises of the 14th, 17th, and 20th Centuries

PART IV: CROSS-CORRELATION OF THE SCALING MEASURES p. 15

One of the major patterns of variability in city distributions is the primate city effect: the primate and top ranked cities often form a steeper urban hierarchy in periods of economic boom or empire, or when the primate city is a major international trade center. In periods of decline they may form a truncated tail compared to the body of the distribution. Further, the slope of the tail of the size distribution (*β*) tends to change faster than the shape body of the distributions. This is tested using data from all three regions using the autocorrelation function (AFC) , in Figure _, where values of a variable in one time period are correlated its values at time lags 1-16 (each lag here is 50 years). The AFC of *β* compared to *q* shows a much higher short term continuity (1 lag of 50 years), a recovery period at 5-6 lags, and then autocorrelation largely disappears, while *q* varies more continuously with more stable long term autocorrelations (up to 16 lags or 800 years). The upper and lower confidence limits are at 95% for a two-tailed significance test (p<.05).



q β

Figure _: Autocorrelation functions of *q* (long, smooth) and *β* (short), all regions.

In comparing *β* and *q* we know they relate theoretically in that if every distribution fitted *q* perfectly, then the relation between them at the asymptotic tail should be $\beta \equiv 1/(1-q)$, as shown in the diagonal line in Figure 7. This is not the case empirically, however, because the *β* for the tail varies independent from the *q* for the body. The plot of their joint distribution shown in Figure 7, where the observations come from all three regions, shows a regression line for actual *β* and *q* that is orthogonal to their relation under the assumption that they fit a common curve with an asymptote of the curve follows a power law where $\beta \equiv 1/(1-q)$. The regression analysis for variable $1/\beta$ with *q* for the historical data gives $\hat{\beta} = 1/(1-.3q)$, with $R^2=.31$ (the regression line is the same for $q>1.22$ and $q>1$, where $\beta \equiv 1/(1-q)$ does not apply). This orthogonality shows the need for both measures, one for power-law tendencies in the top 10 cities that are quite independent of the *q* values that are consistent with a different power-law asymptote.

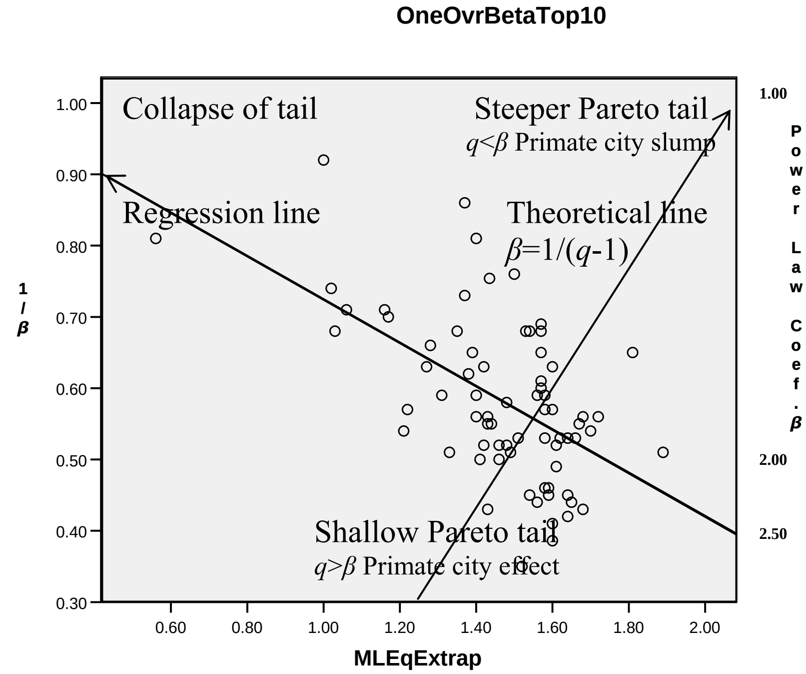


Figure 7: Empirical space of estimated *q* and *β* values

The dispersion of *q* and *β* values within Figure 7, as shown for Figures 8 and 10, is much less for China than in Europe (Figure 8) and Mid-Asia (Figure 10). The urban system of China as a whole, except for the outlier of the crash of 1150, is more stable. The cross-correlations in Figure 9, where each lag represents 50 years, as in the cross-correlation graphs to follow, show for China and Europe that *q* and *β* are positively correlated at one point in time, the lag 0 correlation.

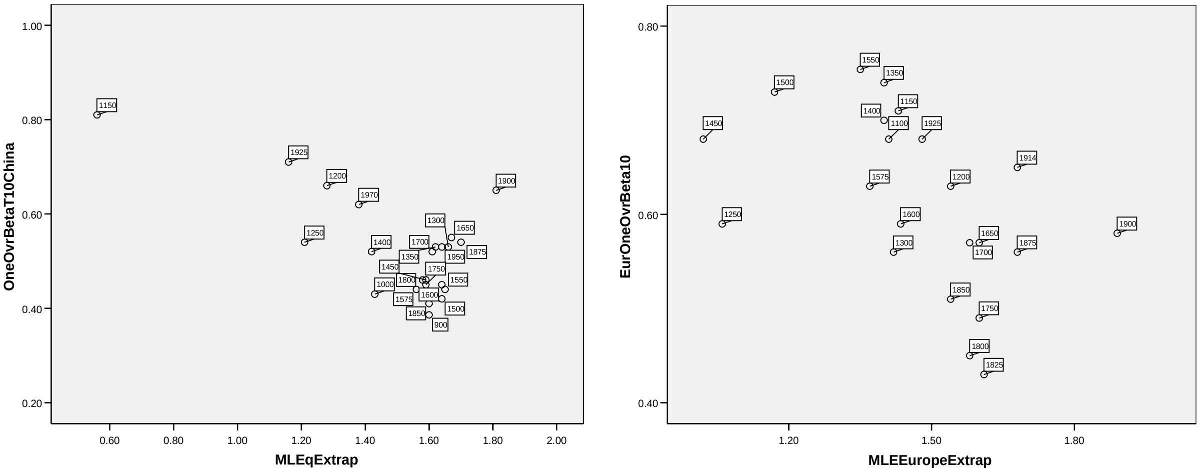


Figure 8: MLE-*q* and Pareto *β* for China and Europe

The question addressed by Figure 9 is whether, in time-lagged correlation: Does the shape (*q*) of the body of the city affects the tail (*β*) in subsequent periods, or the reverse? We have in mind in posing this question that *β* might shape *q* if it is long distance trade has an effect on the larger cities engaged in international trade, but *q* might shape *β* if it is the waxing and waning of industries in the smaller cities that feed into the export products for the larger cities. Figure 9 shows that as a departure from the initial high correlation at lag 0 for China and Europe (but not replicated in Mid-Asia), high *q* (e.g., over 1.5) drives Pareto *β down* over time (often under 2, reducing the slope of the power law tail below that of the Zipfian), which suggests that high *q* produces an urban system decline in *β*. This would contradict an hypothesis of long-distance trade as a driver of rise and fall in the larger cities. It would not contradict, however, the possibility that long-distance trade was directly beneficial to the smaller cities, with these effects feeding into the success of the larger cities with a time lag. For China and Europe, where successful long-distance trade was organized on the basis of the diffusion of effective credit mechanisms available to the smaller merchant cities, this seems a plausible explanation for the time-lag findings. The credit mechanisms that made effective long-distance trading engagements possible in these regions, however, were not so easily available in Mid-Asia where Islam operated to regulate interest rates to prevent excessive usury.

If there is a correlation between long-distance trade and the rise and fall of population pressure in the secular cycles of agrarian empires, our data might support Turchin's (2007) argument, formulated partly in response to our own studies of the role of trade networks in civilizational dynamics, that it is during the high-pressure (stagflation) period that long-distance trade flourishes. If so, the impact of trade should be reflected first in variations in *q*, which vary more slowly than *β*.

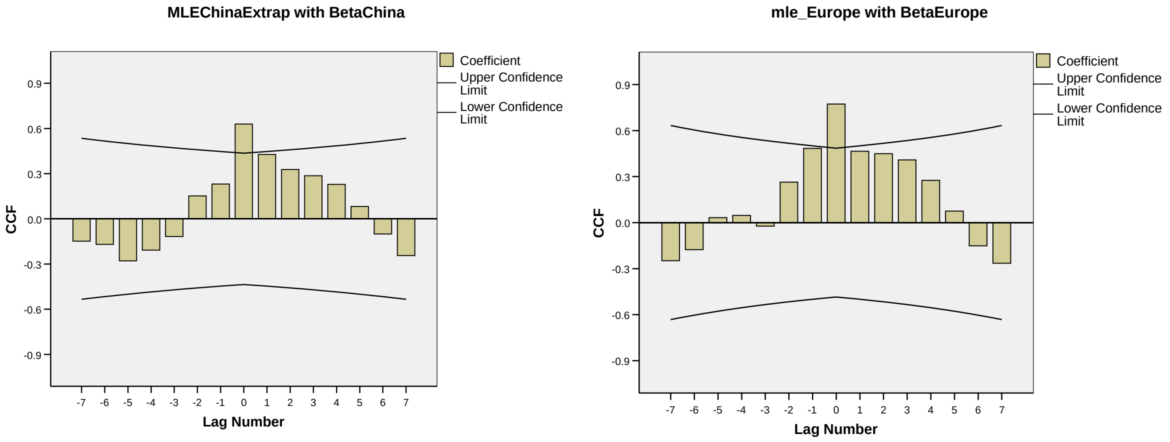


Figure 9: MLE-*q* drives Pareto *β* for China and Europe

Variables q and β do not cross-correlate for Mid-Asia, but detailed examination of the time-lags in Figure 10 shows a weak cyclical dynamic of Hi-q→Lo-β →Lo-q→Hi-β that holds to 1950.

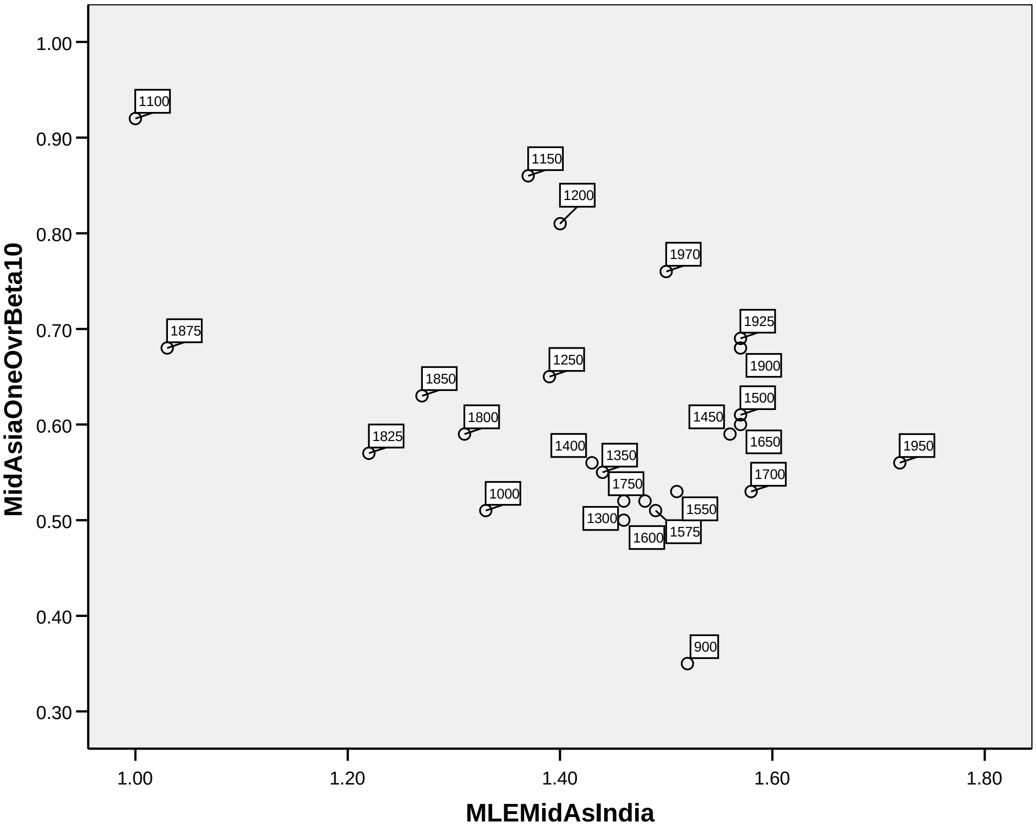


Figure 10: MLE-q interacts dynamically with Pareto β for Mid-Asia, partly reflecting the patterns of Figure 9

The overall pattern for the three regions is shown in the cross-correlations in Figure 11, supporting that of Figure 9. The strong correlation between q and β at lag 1 ($p<.000001$) synchronically gives way to falling values of β over time in the 50-year lags 1-3 in the figure.

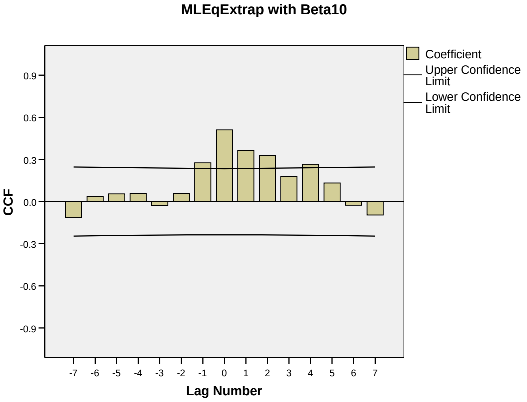
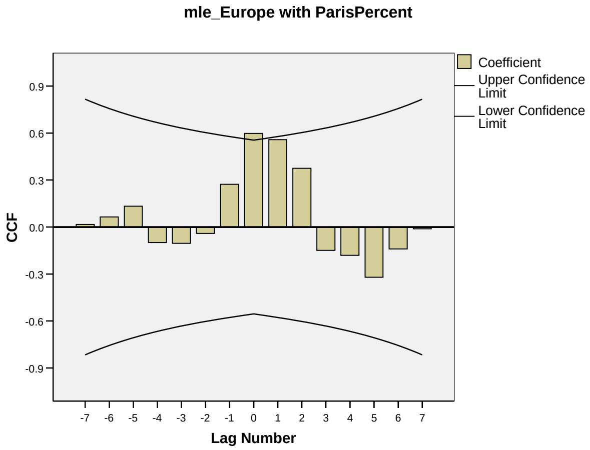


Figure 11: MLE-*q* drives Pareto *β* overall for Eurasian



PART V: HISTORICAL NETWORK AND INTERACTION PROCESSES p. 20

Turchin (2007) shows evidence for "a great degree of synchrony between the secular cycles in Europe and China during two periods: (1) around the beginning of the Common Era and (2) during the second millennium." We also find evidence for synchrony in city system rise and fall in common temporal variations in *q* for the second millennium. The correlations in *q* by time period follow a *s*ingle factor model, as shown in Table 4, with China contributing the most to the 47% common variance between the three regions.

Table 4. Communalities

	Initial	Extraction
MLEChina	1.000	.660
MLEEurope	1.000	.444
MLEMidAsIndia	1.000	.318

Extraction Method: Principal Component Analysis.

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.422	47.403	47.403	1.422	47.403	47.403
2	.944	31.467	78.870			
3	.634	21.130	100.000			

Component Matrix(a)

	Component
	1
MLEChina	.812
MLEEurope	.667
MLEMidAsIndia	.564

Extraction Method: Principal Component Analysis.

The evidence from city sizes adds detail on dynamical interaction to that of synchrony for the last millennium. Figure 12 shows that changes in q for Mid-Asia lead those of China by 50 years (the hugely significant correlation at 50-year lag 1; the lead of Mid-Asia over Europe by 150 years (lag 3) is not quite significant), and those of China lead Europe by by 100 years (lag 2).

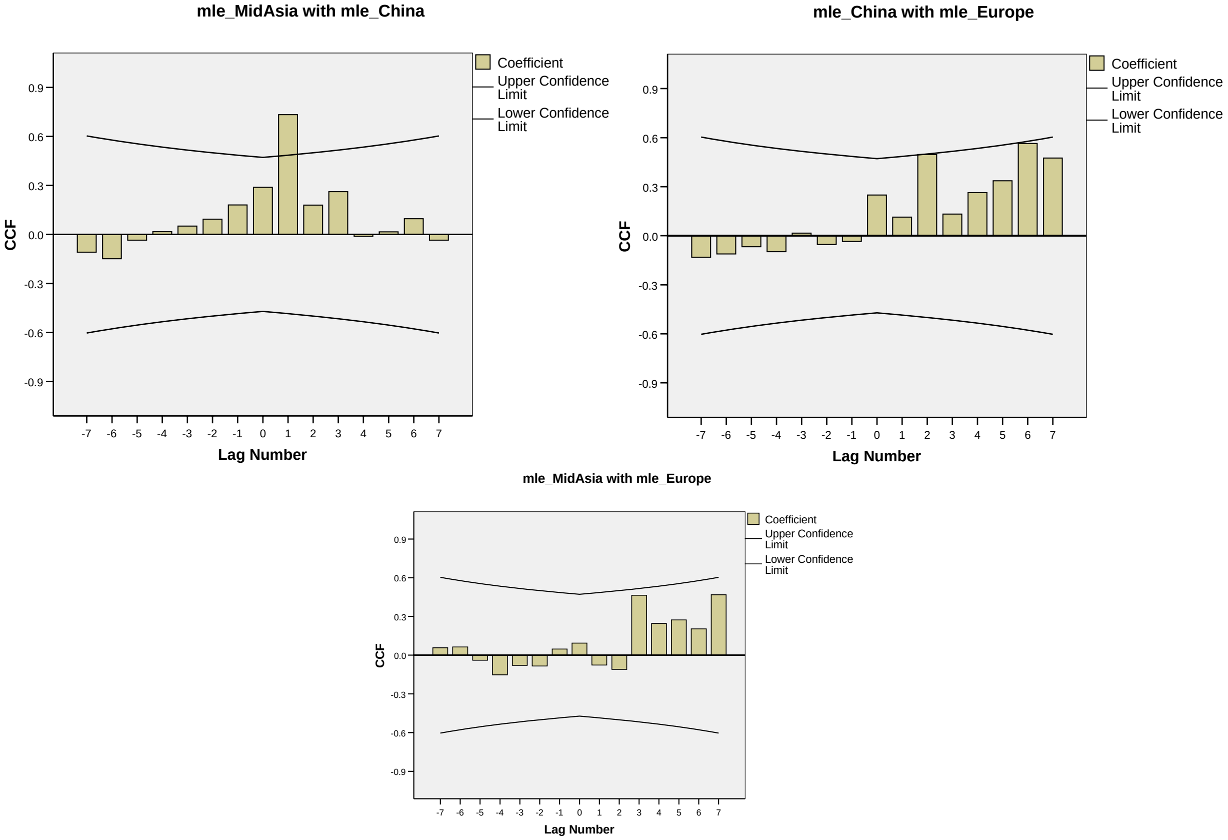


Figure 12: Cross-correlations for temporal effects of one region on another

Our hypothesis has been Eurasian synchrony has been largely due to trade, particularly that between China and Europe. The cross-correlation in Figure 13, for the effect of Silk Road trade on growth of β in Europe, sustained by the Silk Road trade, for example, suggests that trade is indeed what causes the growth of power-law tails in urban size distributions.

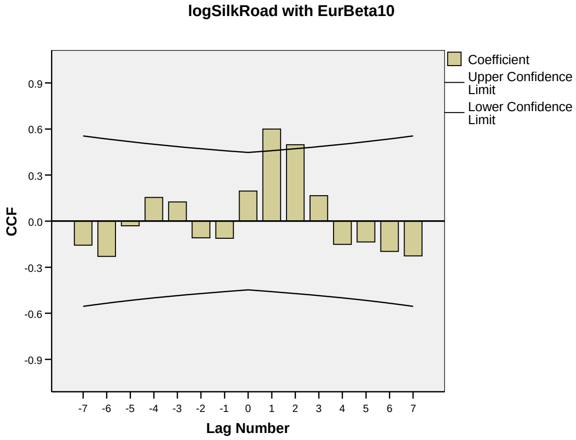


Figure 13: Cross-correlations for time-lagged effects of the Silk Road trade on Europe

Our choice of the last millennium to test the interaction of the city size fluctuations with historical dynamics was motivated by the evolution of globalization in Eurasia in this millennium. Key elements in the transition to market-driven globalization occurred in China starting in the period of 10th century invention of national markets, with currencies, banks and market pricing, a historical sequence that leads, through diffusion and competition, to the global system of today (Modelski and Thompson 1996).

The data on credit and liquidity in the Chinese economy also follows closely the rise and fall of *q*, as shown crudely in Figure 14. Rise of monetization, growth of credit, and development of banking accompany the early Zipfian q~1.5 of Song China, and these mechanisms of liquidity plummet with the Jin conquest of Kaifeng. Circa 7-800 years from 1100 BCE, with long periods of inflation, are required to regain liquidity and banking favoring international trade. During the Qing dynasty the Chinese money was silver coin. The first modern bank, the Rishengchang (Ri Sheng Chang) was established in 1824. It broadened to include banks in every major city, folding in bankruptcy in 1932. (the right end of the liquidity graph, estimated qualitatively in the lower figure, should be higher).

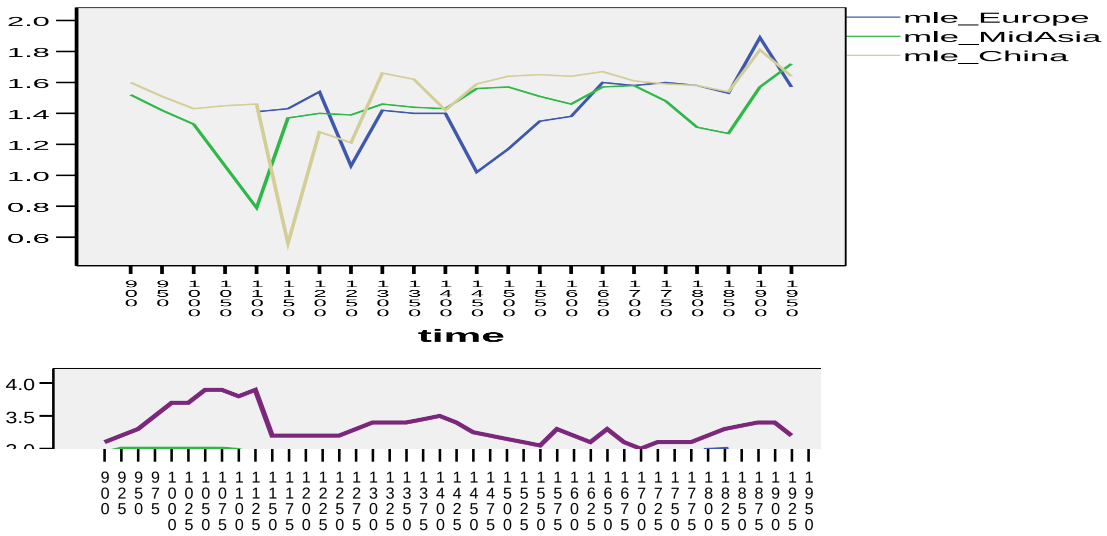


Figure 14: A crude long-term correlation between Chinese credit and liquidity and *q* (REDO)

We have scant data on total population relative to resources, and we have reliable data for the last millennium only for England in comparison with our Eurasian city data. There are few points of comparison, but temporal synchronies appear in those few points: 1300 and 1625 are the peaks of scarcity for ratios of people to resources and pre-1100, 1450, and 1750 are the troughs of plentiful resources. The peaks correspond to slumps in *q* and the troughs to rises.

It is impossible to rule out at this point the possibility that our urban system fluctuations are interactively linked to Turchin's secular cycles, particularly if we include both types of fluctuations: those in *q*, in *β*, and in our normalized minimum of the two, as wall, which reflects either type of slump.

For China, the 800 year lag between urban system collapses in *q* resemble J. S. Lee's interpretation of 800-year cycles of internecine conflict in China, as shown by his data, reproduced in Figure 15. There is also a dip in *β* in the middle of this period that might indicate shorter major fluctuations of the Chinese city system. [9] Mid-Asia also shows 7-800 year lags in our data between urban system collapses, and also has some dips in *β* in within the longer period.

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[9] In some of our earlier analyses there were tenuous indicators of 400-year periods and possibly 200-year periods of city size oscillation that seemed to correspond more closely with Turchin's secular cycles. With more accurate MLE measures, however, we see longer periods of stability, and we attribute the appearance of shorter oscillations in the earlier analysis to biased estimations that, in introducing intermittent error, tended to break longer periods of stability into what appeared to be shorter ones.

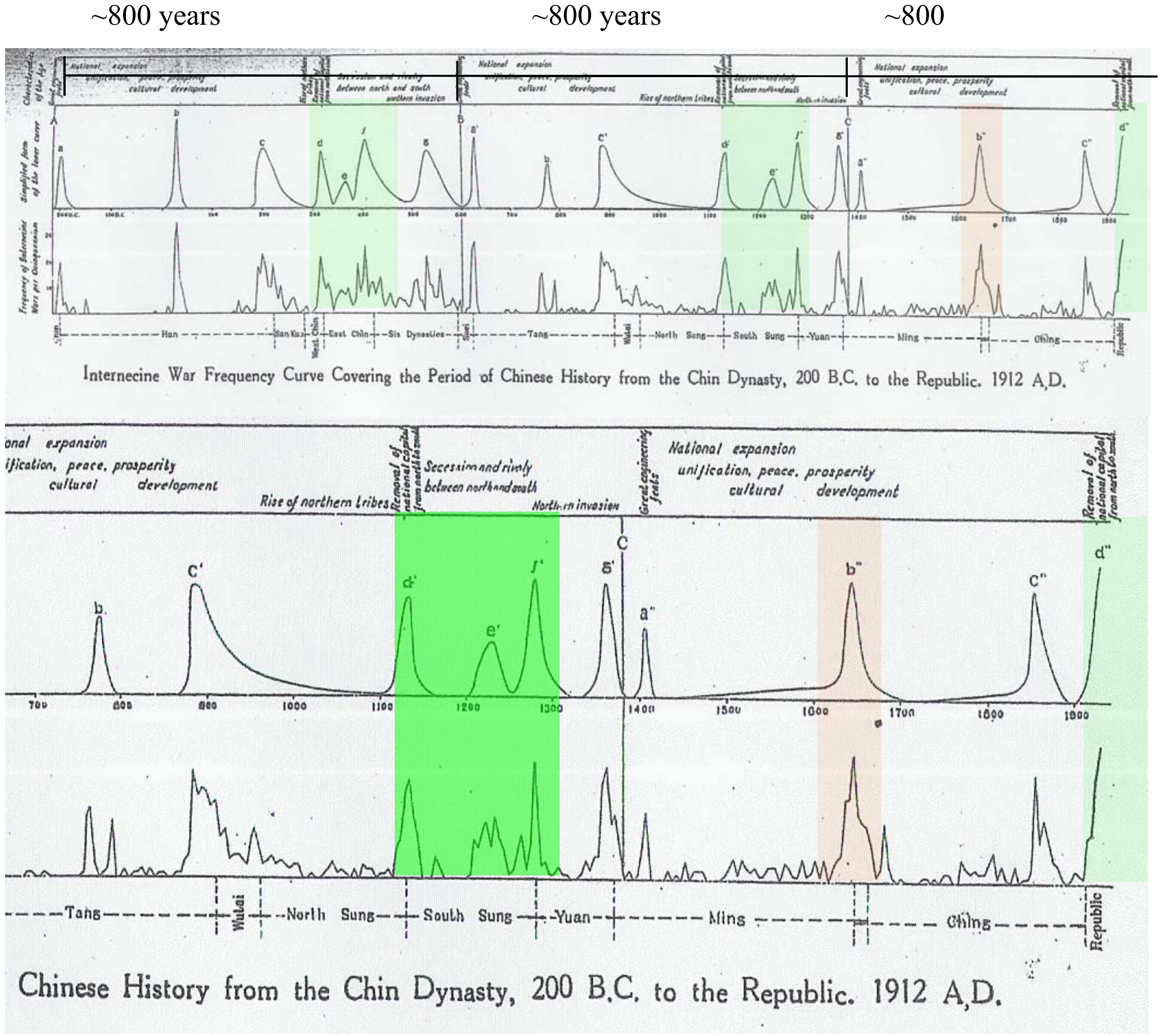

Figure 15: Long-cycle internecine war cycles in China

PART VI: CONCLUSIONS p. 24

This study and those that preceded it began as experiments in building from two sets of sources, one in quantitative history and the work of Goldstone (1991), Nefedov (1999), Turchin (2003), and Spufford (2002) and the other in mathematical and measurement concepts that incorporate city size distributions as an object of study in a more meaningful way. The development of unbiased estimates of variations in city size distributions using maximal likelihood (MLE) allowed a level of precision and accuracy even with small samples that led to some useful findings in this study that we think are reliable.

By focusing on the 75 largest cities over a series of time periods that go back to antiquity – closely enough spaced to get the quantitative variations of the full cycles of city system oscillations – Chandler makes available a full run of data for studying how city system evolutions couple with agrarian sociopolitical dynamics. Initially, this will not be equally possible in every region at every time period, but only where the density of cities is sufficient for quantitative study. By focusing on China, however, we availed ourselves of one of the richest pockets of Chandler's comparative data on cities, especially for the early period of globalization, where China had the largest number of large cities. In refining the present model for comparison with other regions we will consider whether to adjust for Chandler's possible underestimation of walled city populations for China, but the biasing assumption he made for China's walled cities (Appendix A) does not carry over to other regions.

We did find strong evidence of historical periods of rise and fall in the city systems of different regions, and time lagged effects of changes in city size distributions in one region on other regions. These are weak and slow from Mid-Asia to China, and strong and fast from China to Europe, which makes sense in terms of the Silk Roads trade. Perhaps this is the missing evidence of synchronies that Chase-Dunn, Niemeyer, Alvarez, Inoue, Lawrence and Carlson (2006) were looking for but with cruder comparisons. Most of the correlations, however, are time-lagged rather than temporally synchronous. The effects run in the directions suggested by Modelski and Thompson (1996).

We are reasonably confident in concluding that the Pareto I and II (*q*-exponential) measures of city hierarchies through time, especially when used in combination, can provide a measurement paradigm of standardized methods and tests of replication in historical comparisons. The attractive features of the *q*-measure gain added benefit from the precision of our measures with the use of MLE method.

Richness of supporting data would logically take us next to Middle Asia and the larger world from 700 CE that embraced the rise of Islam, the Mongol use of the Silk Roads and development of new towns and cities on those routes to link China and the rest of Middle Asia into a global system. Such a study, modeled on this one, would include the role of the Indic subcontinent, and that of the Mongols (Barfield 1989, Boyle 1977) in trade and conquest, the Arab colonization of North Africa and Spain, and the feeding of urban developments in the Mediterranean, Russia, and Europe.

When we compare our results, measurements, and mathematical models to those of the structural demography or secular cycles studies of Goldstone (1991), Nefedov (1999), and Turchin (2003, 2005), we find a novelty that separates our findings from that of the standard Lotka-Volterra oscillation model for historical fluctuations. Turchin (2003, 2004), for example, argues the Lotka-Voltera dynamic works optimally when one of the interactive variables (say, population/resource ratio measure of scarcity and sociopolitical violence) is offset by ¼ cycle. Our cycle of city-size oscillations might be four times as long as Turchin's secular cycles (J.S. Lee divides his 800 year periods into two periods of 400, seeming to do with an early growth of early forms of "empire" in a region, then a time of turbulence in the second period; then a new cycle of empire. It is possible that the city cycle operates at this level, at the larger civilizational networks of states alternating with forms of empire. Long city-size system oscillations of ca. 800 years would not be offset by a ¼ cycle but by ⅛th of a cycle, which is a long period of instability (vulnerable to conquest from the outside following internal instabilities). From our perspective, however, sociopolitical instability is not smoothly cyclical but episodic. Rebellions, insurrections, and all sorts of protest are events that mobilize people in a given time and generation, and that impacts that, when repeated frequently, have massive effects. We see this in long-term correlations with SPI, such as internecine wars in China.

We have been able to discern some of the effects of trade fluctuations (if not trade network structure) in these models. The monetary liquidity variable for China, in one of our tests, showed the effect of a trade-related variable on q. We think it possible to reconstruct trade routes as historical time series, and to do ordinal ranking of trade volumes on these routes. We think that these have strong effects, along with disruptive conflicts and political or empire boundaries, on the economies of individual cities and regions, and that these variables could be shown to have dynamical interactions within the context of secular cycle and urban systems rise and fall.

We can be less sure about inferring from empirical results for China or Europe to the real world of Chinese and European history and forward-looking prediction, because we expect to be able to do even better estimations with derivatives, ML estimation, and evaluation or correction of biases in a reconstruction of Chandler's China dataset. Our hypotheses must remain hypotheses, not yet rejected. Some of patterns we to see in our data as concerns globalizing modernization are consistent with prior knowledge and others are startling. To be expected are the developmental trends of scale – larger M (largest cities), larger Y0 (total urban population), and larger P (total population). With time, the crossover to power-law parameter (Pareto II "scale" or σ) moves further down the tail, so that more and more of the city distribution becomes power law, consistent with much of the previous work on power-law scaling.

What is startling is that there might be are long-wave oscillations in *q* that are very long. Hopefully, a long-term trend and contemporary structure of Zipfian city distributions is an indicator of stability, but even the 20th century data indicate that instabilities are still very much present and thus likely to rest on historical contingencies (somewhat like the occurrence of a next earthquake larger than any seen in *x* years prior), and very much open to the effects of warfare and internal conflicts that are likely to be affected by population growth, and as opposed to stabilization of trade benefiting per-capita-resources ratios.

Our results allow us to consider the edge-of-chaos metaphor of complexity with respect of *q* as a first but largely insufficient approximation to an explanation for what we see historically. It is a truism to say that complexity, life, history, and complex systems generally stand somewhere between rigidity at one pole, which might be exemplified by $q>1.7$, and on the other, an exponential random distribution ($q \approx 1$) of city sizes, or the heightened unpredictability of chaos ($0<q \ll 1$).[10] But we do not see support for *equilibrium* on the "edge of chaos" in these data. The historical *q*-periods of China and other regions tend to cluster, somewhat like "edging on chaos," near an average of $q \approx 1.5$, but they do so in terms of oscillations, not far from equilibrium, but not a stable equilibrium. From that average they may fall into decentralized chaos, here in the metaphoric sense (but with exceptionally low *q*), in which the power-law tails are absent (with larger cities seemingly crushed in size by internecine wars) and smaller cities are frequent relative to hubs, or rise into regimes are affected by massive external drains on the economy or political policies that seem to put *q* into abnormally rigid states (exceptionally high *q*). The directions of change in *q* are largely predictable as a function of the current-state

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[10] Technically, the mathematics assigned to chaos is a deterministic departure from randomness in which a dynamic trajectory never settles down into equilibrium, and small differences in initial conditions lead to divergent trajectories. The link between empirical history and "edge of chaos" is typically done by simulation.

variables (such as population/resource ratios and sociopolitical violence) in the historical dynamics models up to, but not yet including, the contemporary period. How to derive predictions for the contemporary era is not yet evident given the new configurations of industrial societies, but it is very probable that such predictions as do emerge for the present will contain processes operative in the past.

APPENDIX A: Chandler's Chinese City Data.

Many Chinese historians question the estimates of Chinese city population densities given by Chandler (1987:7): "Chinese cities tend to have an especially low density because of the Chinese refusal to sleep below anyone, so their houses are of nearly all of just 1 story. Hence, inland Chinese cities had a density of only about 75 per hectare, and even in seaports or the imperial capital the density hardly exceeded 100." One such anonymous reviewer considered this an underestimate by half based on current knowledge.

To test the effects of these likely underestimates population, we examined the data from the first of our periods, as shown in Table A.3, after checking Chandler's (1987:417-451) text to note the basis for his estimates for each city. We adjusted the five city estimates affected by raising each by 50% (half the attributed error), recomputed the number of cities for each size category, and then compared the power-law coefficients for the original and the revised data as well changes in estimates of *q*. Changes were evident and significant in each case. They did not change the r^2 for goodness of fit, but for *q*, for example, the value change from 1.9 to 0.7. Similar tests can be made for other periods, but we may need independent evidence as to the extent to which the critique merits new density computations.

Possible errors of this sort in Chandler's estimation assumptions, then, could change our scaling results. These biases could change the patterns of variation in *q* seen in our results. Because Chandler uses the common assumptions throughout these historical periods up to 1950, however, it is possible that they may not affect our historical comparisons about relative changes in *q*, at least up through 1950. For 1962, 1968, and 1970 he has taken the city population data of Richard Forstall of Rand McNally and Co. To test the robustness of our results, however, would entail a project that builds on the Excel spreadsheet for the Chandler data as exemplified above. By consulting his book to pick out each variable used to make multiplicative estimates – such as size of urban area, number soldiers (e.g., times 6), number of streets, etc. – a spreadsheet of calculations with the multiples and their base numbers might be useful to improve Chandler's estimates using a Bayesian weighting by consistency.

Chandler (1987:6) also notes that "the large growth of suburbs outside city walls had not begun before 1850 except in the newly rising industrial conurbations of Britain." This might cause problems "since the presence of suburbs has been well documented in history with new walls built to enclose a population that had spread beyond the original walls" (Pasciuti and Chase-Dunn 2002:1). Chandler does include suburbs in his criteria for city boundaries, however, and in his estimates throughout the book.

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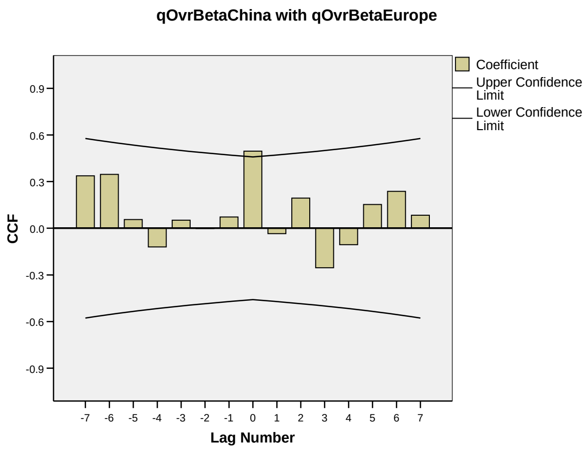
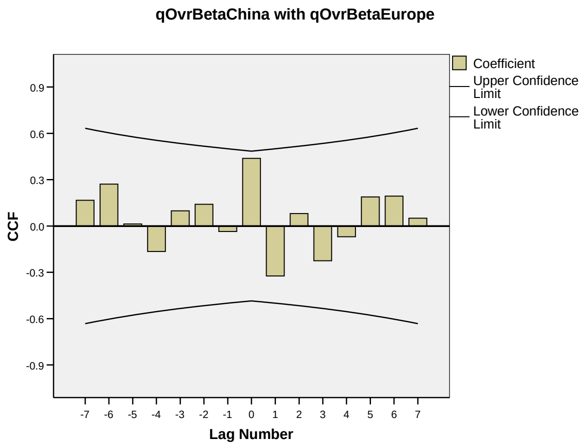

qOvrBetaChina with qOvrBetaEurope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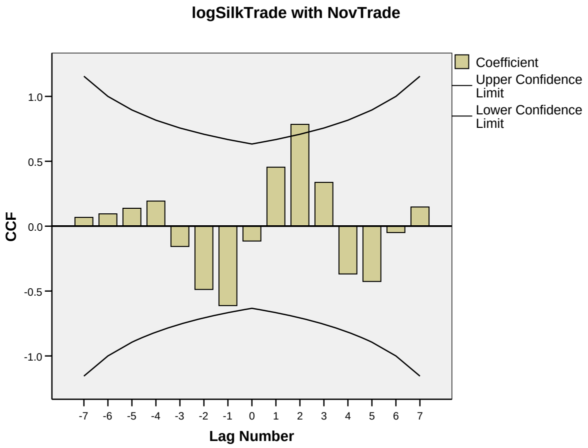
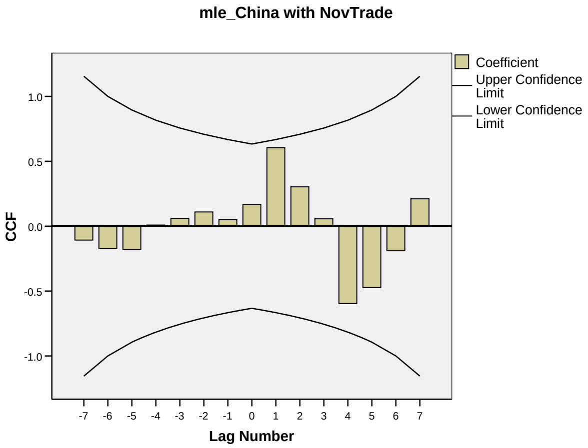
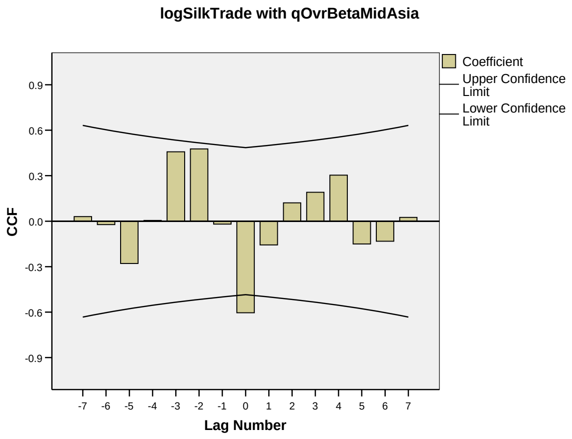
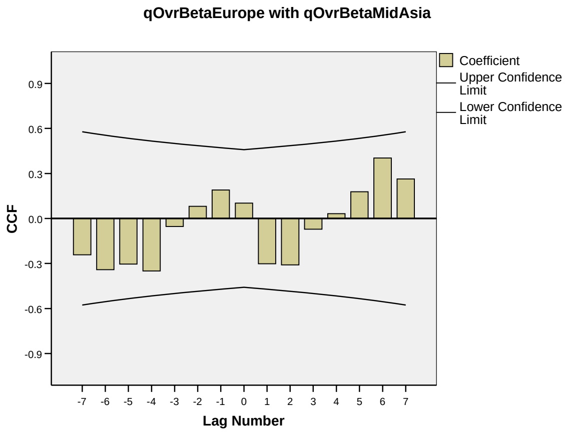
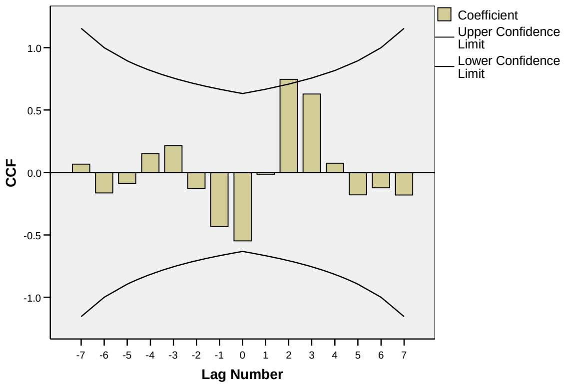

logSilkTrade with NovTrade

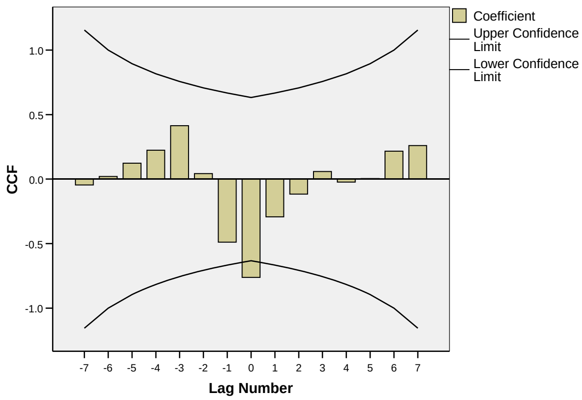

mle_China with NovTrade


logSilkTrade with qOvrBetaMidAsia

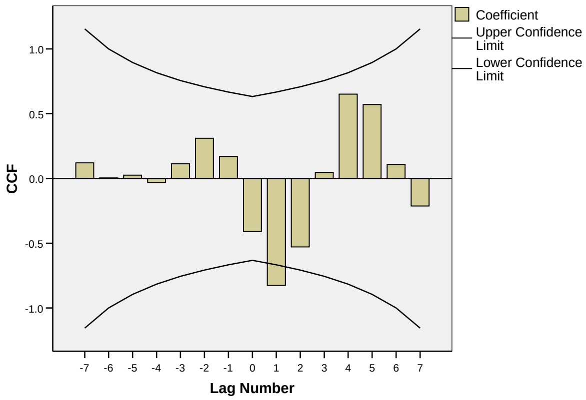

qOvrBetaEurope with qOvrBetaMidAsia



qOvrBetaMidAsia with logSilkTrade



qOvrBetaMidAsia with mle_China



qOvrBetaMidAsia with NovTrade

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Figure 9 shows the autocorrelation functions of *q* for the three regions. Europe is the most cyclical (as reflected in the runs tests), China less so, and with Mid-Asia showing only 50-year stabilities. The cycling patterns grow weaker in these cases, but the cycling time seems to weakly approximate 800+ years (unit intervals in these graphs are 50-year increments).

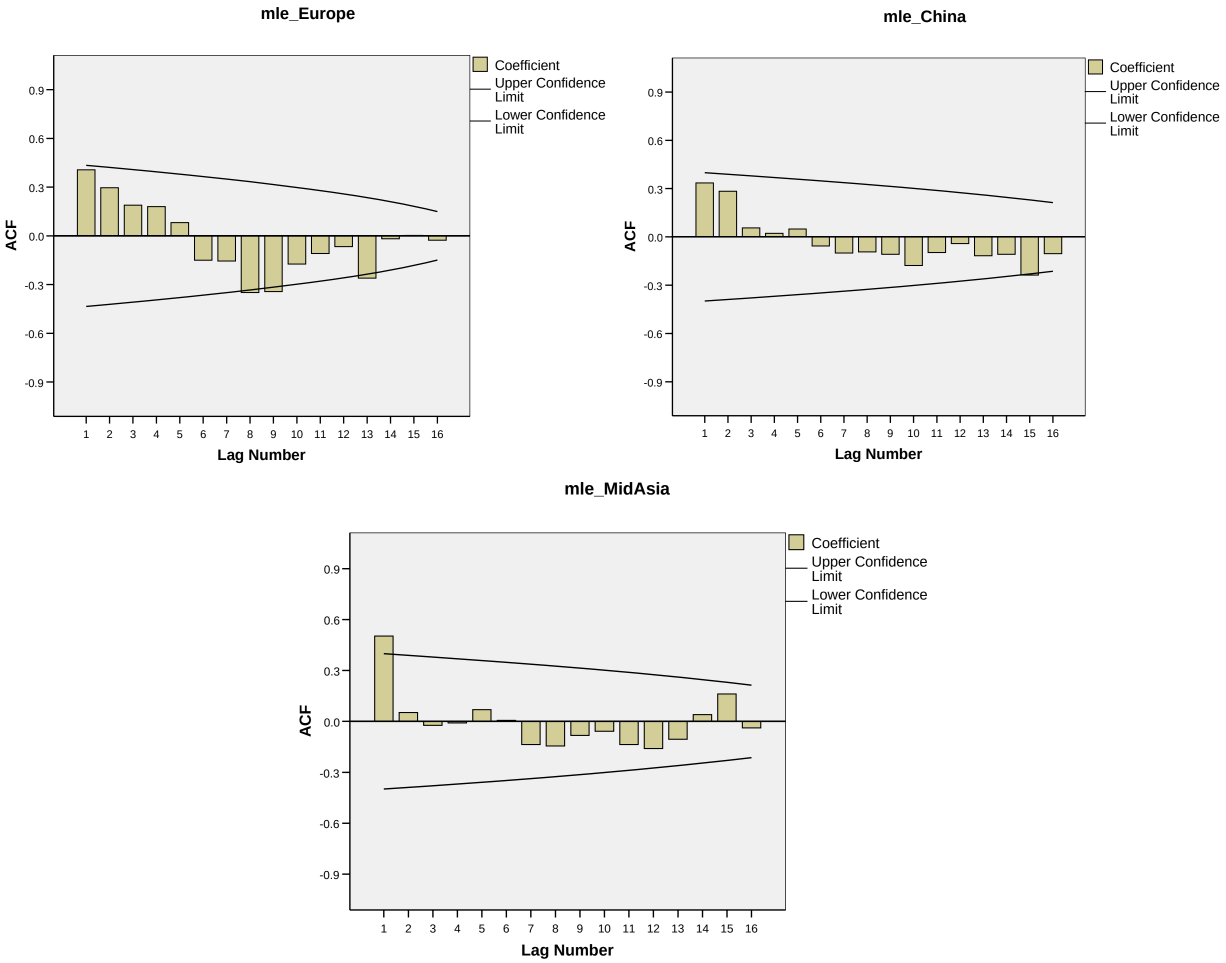


Figure 9: Autocorrelation functions for the three regions

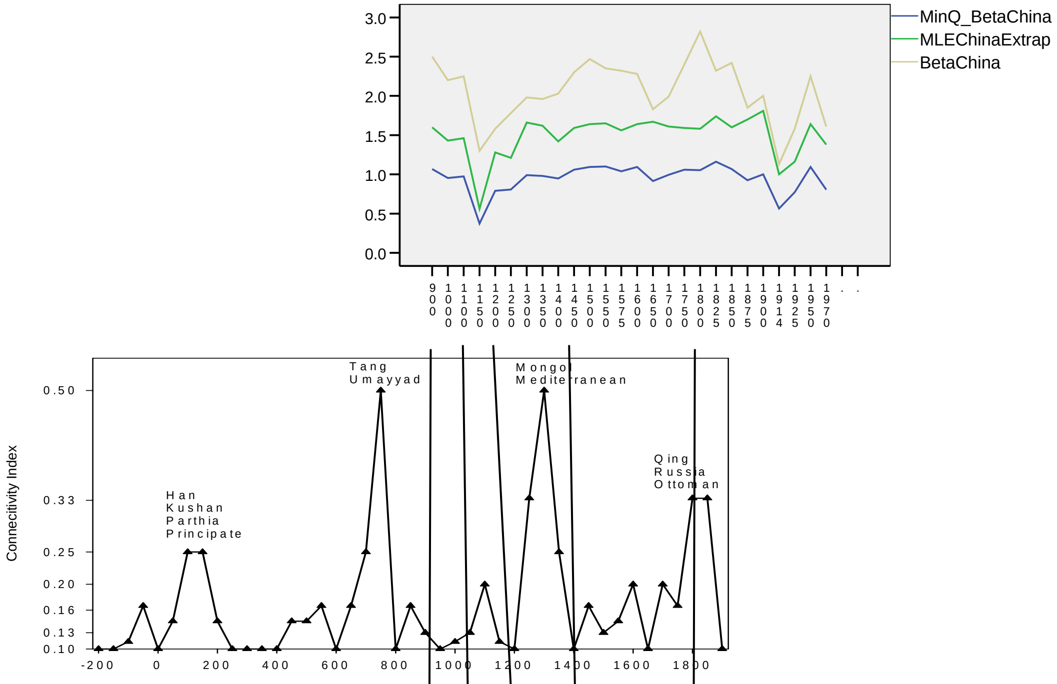
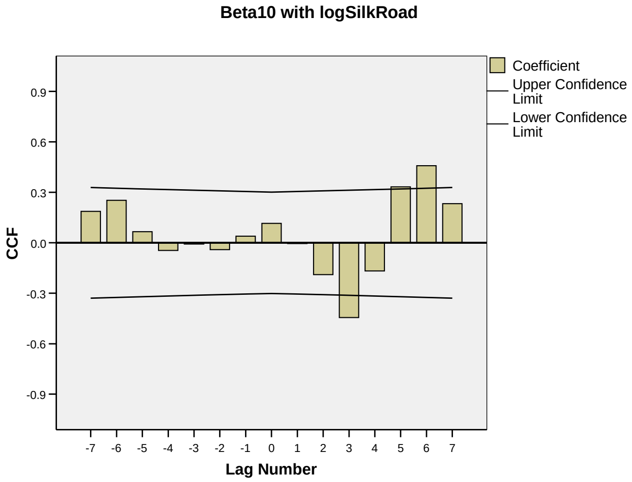


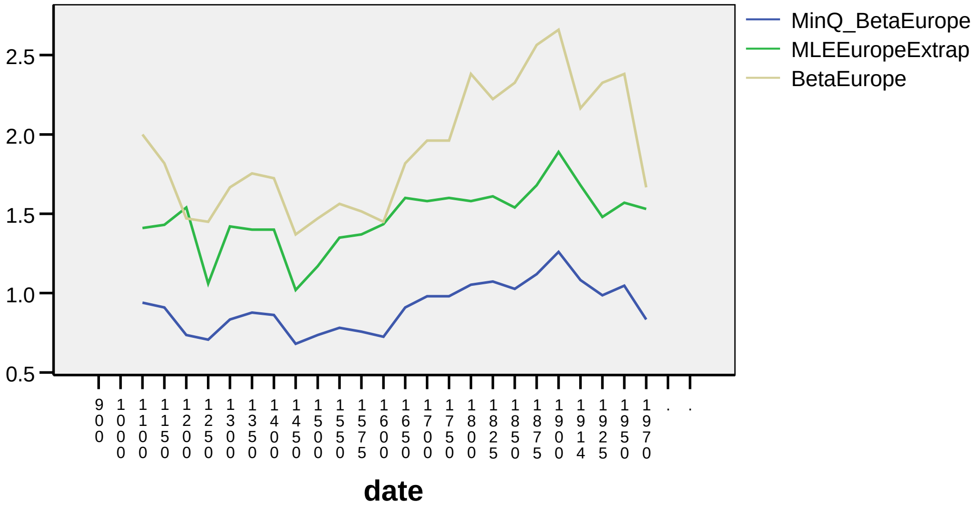
Figure 10: Silk Routes Connectivity

Method: Tsal.fit. Takes the rank-size city distribution: *q* and κ are now uncorrelated. This is good.

To fit a power law, Spss needs the cumulative distribution. Easiest way to do this in the Chandler file, sorted by period and size, is to create a second column to the right of the size column, and make it a cumulative sum for all cities. Then go to each largest city turn this row yellow, and pull over the largest city size. Then Paste the series for one period into a column of Spss. ChandleCumPop.sav has CumPol by rank. Create a variable called rank from 1 to n. If you want only the top cities make n=10 (TopTen). To compute the Pareto 1/Beta coefficient in Spss/Analyze/Regression/Curve Estimation [x] Power law. This gives Beta=alpha+1 where alpha=1 is Zipf. To do the whole series with [x] Display Anova table, select “Top10” as the independent variable and enter all the regionYears (e.g., “c1200”) as dependent variables (run them all at once. Then when you enter the results in a spreadsheet do a transformation on 1/Beta to calculate Beta as it’s reciprocal, e.g. .5 becomes 2.0, the Zipfian Beta. For a single period select “Rank” as the dependent variable and cYear (e.g., “c1200”) as the Independent Variable and you get Beta directly.

CHINA and MID-ASIA: indirect effect of conquest?


Beta10 with logSilkRoad
Coefficient
Upper Confidence Limit
Lower Confidence Limit
CCF
0.9
0.6
0.3
0.0
-0.3
-0.6
-0.9
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
Lag Number


MinQ_BetaEurope
MLEEuropeExtrap
BetaEurope
2.5
2.0
1.5
1.0
0.5
900 1000 1100 1150 1200 1250 1300 1350 1400 1450 1500 1550 1575 1600 1650 1700 1750 1800 1825 1850 1875 1900 1914 1925 1950 1970 . .
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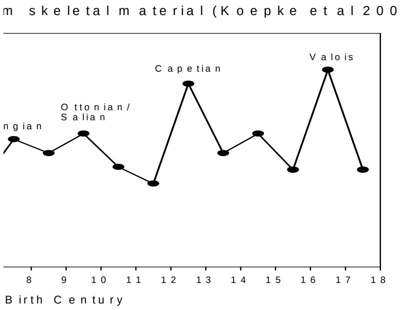
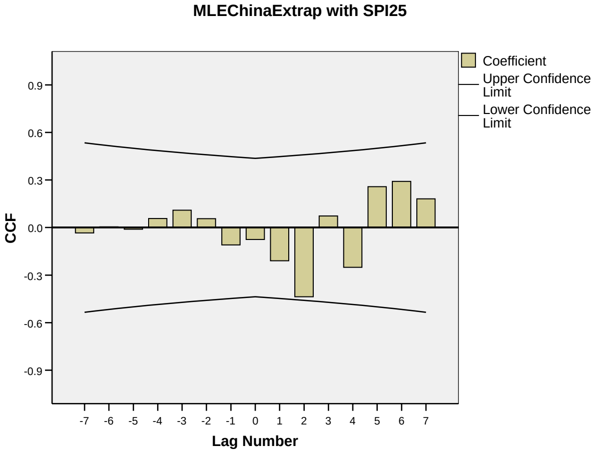
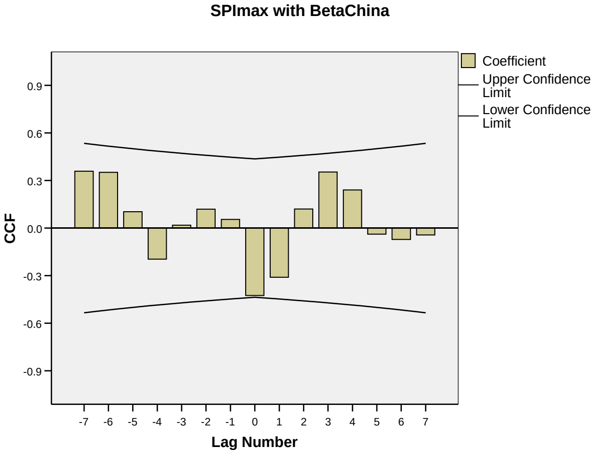

m skeletal material (Koepke et al 2005)
ngian
Ottonian/ Salian
Capetian
Valois
8 9 10 11 12 13 14 15 16 17 18
Birth Century

opposite to predicted: China q Up SPI down! As predicted: China

Does SPI(max) at LOW beta and then predict beta recovery? (possibly)



MLEChinaExtrap with SPI25



SPImax with BetaChina