World Population: Trends, Mechanisms, Singularities [1]

(References contain bolded blue live html links)

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World demographic transitions are epochal events in human evolution. To understand both the enormity and the subtlety of transition that humanity faces today, it is helpful to have the variety of temporal and theoretical perspectives provided by archaeology, demography, formal modeling, and interdisciplinary research—anthropological, political and economic, and mathematical—on world historical configurations. Demography proper has tended to lose itself of late within a micro level of analysis, in the vain hope that understanding behavior at the individual level will provide the understanding needed for larger problems. As Johansen and Sornette note (2001:2):

> Demographers usually construct population projections in a disaggregated manner, filtering the data by age, stage of development, region, etc. Disaggregating and controlling for such variables are thought to be crucial for demographic development and for any reliable population prediction. Here, we propose a different strategy based on aggregated data, which is justified by the following concept: in order to get a meaningful prediction at an aggregate level, it is often more relevant to study aggregate variables than "local" variables that can miss the whole picture in favor of special idiosyncrasies.

The ability to study the interrelated aspects of growth theory, macropolitics and macroeconomics that emerge from interactions in markets and other forums, for example, may provide a much needed antidote. Formal demography (Lee 2001) may provide an analytic link between events at a macro level that pertains to populations and world systems and individuals and their behavior at the micro level. We show here the importance of a larger temporal, interdisciplinary and world perspective.

Formal Definitions of Growth Trends and Transitions

Demographic transitions are defined by changes in *growth trends*. There are two types of models of population growth trends, *endogenous* and *exogenous*. We will consider these after we complete the discussion of endogenous trends. An *exogenous* trend is one defined by a function for population change, $dN/dt = f(N, N^2$, and other powers of N, plus other variables), that is, a function of various parameters for N, powers of N, and any number of exogenous variables and their exponents and parameters. Such functions may include interactions among different variables. In an *endogenous* model, population change is often considered to be a function of various parameters for N and N^2, so that $dN/dt = f(N, N^2$, and other powers of N). The most common functions are of the form $dN/dt = a_0N^r$ where r defines a power law in which $r = 0$ implies *no growth*, $r = 1$ implies *linear growth*, and $r = 2$ *quadratic growth*, as in (1) ad (2):

$dN/dt = a_0N$, expressed over time $t = 0, 1,\ldots, T$ as $N_t = N_0r^t$ and (1)

$dN/dt = a_0N^2$, expressed over time to singularity $\tau = \tau_{max}, \tau_{max}+1, \ldots, 0$. (2)

Equation (1) is read like compound interest, where $r > 1$ is a growth parameter (e.g., 1.01). If growth is compounded continuously, as in certain types of savings accounts, $N_t = N_0e^{rt}$. In either the discrete or continuous case, equation (1) generates an *exponential* growth curve. Equation (2) differs in that time is measured as the number τ of time intervals to singularity τ_0. In this case, if growth is compounded continuously, the number N goes to infinity. In either the discrete or continuous case, equation (2) generates a *power-law* growth curve. Thus, endogenous population trends may be either exponential or power-law, while hyperbolic growth is the special case of the power law $dN/dt = a_0N^r$ where $r = 2$.

In exponential growth, doubling of population occurs in uniform intervals of time (the doubling time of something growing exponentially is 70 divided by its annual growth rate). In a power-law curve, the doubling time occurs in successively shorter intervals, decreasing with the log of time elapsed. Because the doubling intervals become increasingly shorter until a limit is reached at which population doubling recurs in intervals that approach zero at a finite time, power-law growth entails a singular end-point in time that is undefined or, in the continuous case, infinite in its limit. The time at which this occurs is a *singularity*, and the acceleration of doubling keeps pace with the log of time-to-singularity. Physicists use such singularities in processes of power-law growth to predict phase transitions. No physical, chemical or biological growth process can sustain power-law growth up to its point of singularity. Accordingly, physicists can say with confidence that power-law growth, as a universality class, provides an explanation for predictable and observable phase transitions.

Equations (1) and (2), in discrete time, yield respective equations for changes in the population N over time:

$N_t = N_0\, r^t$, where time counts from $t = 0$ forward and $r = 1$ for no growth (3) Exponential

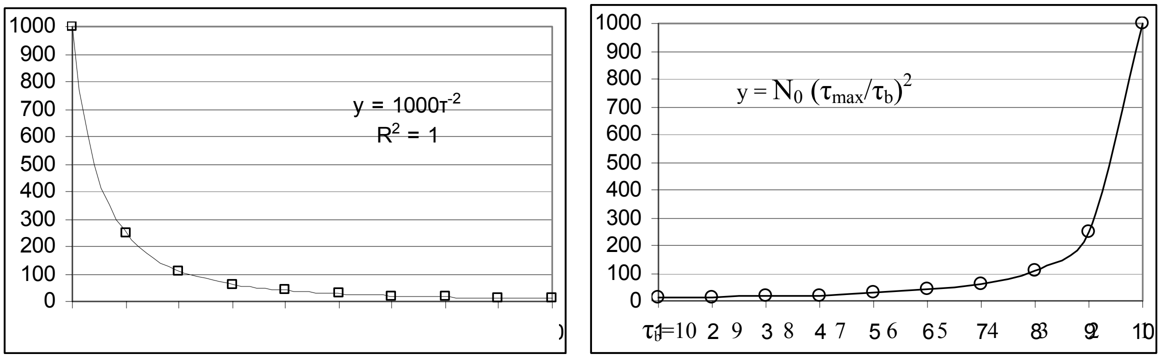
$$N_b = N_0\, (\tau_{max}/\tau_b)^r \qquad (4)$$

Power-law

$$= K(\tau_b)^{-r} \qquad (5),$$

where time τ_b counts backwards from a maximum of $\tau = \tau_{max}$, forward to the $\tau = \tau_0 = 0$ of singularity. The large-population constant, $K = N_{min}\, \tau_{max}{}^k$, derives from (4) and (5). There are four parameters involved in fitting a theoretical to an empirical curve: starting and singularity dates, k and either K or N_{min} in the empirical vicinity of the population size at the starting date. The exponential, by comparison, has three, as is fitting a special case of the power law $dN/dt = a_0 N^r$ where r is set to 1. What we will show later is that the curve-fitting involved is not a simple matter even for endogenous models of well-behaved population growth, even though it may be robust within limits. Because of this, some models of power-law growth prefer to set $k = 2$ and solve for the other three parameters that give best fit to an empirical curve, calling this the hyperbolic growth model. Often there are other reasons to justify on theoretical grounds setting $k = 2$ when this entails the dimension of a field of interaction among N elements (i.e., N x N pairs).

To illustrate power-law growth curves, figure 2a has a large-population constant K and an exponent k for a hyperbolic growth equation, $N_t = Kt^{-2}$, where $t = 1, 2, \ldots, t_{max}$ is an equal-interval time scale to t_{max} one unit from singularity. N shrinks as we move back in time. In the figure, $K=1000$, $\tau_{max}=10$, $k=2$, and $K = N_{min}\, \tau_{max}{}^2 = 10 * 100$. Figure 2b reverses the values in the x axis to show $N_b = N_{1+max-\tau} = N_0\, (\tau_{max}/\tau_b)^{2} = 10\, (10/\tau_b)^{2}$.



(a) Time numbering toward singularity at 11 ***(b) Time in reverse numbering to singularity at 0***

Figure 2. Power-Law Growth in forward time (t) and backward from singularity (τ)

Variants of Endogenous Population Models

An *endogenous* trend is one defined by a function ($dN/dt = f(N, N^2$, and other powers of N$)$), typically of the form $dN/dt = a_0N^r$ and hence $N_b = N_0 (\tau_{max}/\tau_b)^r = K(\tau_b)^{-r}$. Table 1 captures the features of the major endogenous trend models, where a_0, r, N_0, K, and τ_{max} are constants and the only variables are N_b and τ_b. Table 1 utilizes an easy way to represent the eleven possible types of growth regimes in terms of constants a_0 and r, a start date N_0 and for $r > 1$ a singularity date τ_{max} from the start date are also implied. The critical value for type of growth trends is the change in the scale of the growth rate r with power growth in N. Where $r < 0$, the trend is convergence to a stable population, which also occurs where $a_0 = 0$. When $r = 0$, population changes uniformly by constant increments so that growth decelerates but decline accelerates. When $0 < r < 1$ there is a second order polynomial growth ($a_0 > 0$) or decline ($a_0 < 0$) where $N = b + c\,t + d\,t^2$. When $r = 1$, change is exponential, with rises ($a_0 > 0$) starting slow and ending with rapid growth (which will eventually exceed a Malthusian limit to growth), but declines ($a_0 < 0$) start fast and end with rapid extinction. When $r > 1$ we have either power-law growth ($a_0 > 0$) with singularity or power-law decay ($a_0 < 0$) with a long tail and thus a long delay to extinction. Very few of these trends are evident in world population change, but more of the declining types would be evident if we considered regional populations.

Regime	Growth ($a_0 > 0$)	Zero Growth	Decline ($a_0 < 0$)	Regime
$r < 0$ Convergent	Converges up to N*	$a_0 = 0$	Converges down to N*	$r > 0$
$r = 0$ Fixed Change	Decelerating	$a_0 = 0$	Accelerating	Init. Slow Extinction
$0 < r < 1$ Explosion	2nd Order Polynomial Rise	$a_0 = 0$	2nd Order Polynomial Fall	Plunge
$r = 1$ Malthusian	Exponential Rise	$a_0 = 0$	Exponential Decay	Rapid Extinction
$r > 1$ Singularity	Power-Law Rise	$a_0 = 0$	Power-Law Decline	Slow Death

Table 1: Variations of $dN/dt = a_0N^r$

Substantive Definitions of Growth Trends and Transitions.

In adopting a world and evolutionary perspective, we need first to define some of the relationships of world demographic transitions and the more localized transitions in the modern or premodern eras. We will consider a *world demographic transition* as a shift between two relatively constant long-term statistical trends, each described by one of the cells in table 1, but most commonly by exponential or power-law growth curve. Excluding exogenous variables, a shift may occur either in the type of endogenous growth function approximated by the trend or in the growth constant of its functional type of growth (i.e., within one of the cells of table 1), whether exponential or power-law.

In exploring the relation between world population and regional demography, it is essential to note that when time series data are aggregated from very localized regions, where every locale might have exponential growth curves, or there might be a mix of exponential and power law, the aggregate growth curve will often approximate a power-law curve (Farmer 2005). If, for example, exponential growth starts at an initial date in populations of unequal sizes, different regions can be expected to undergo demographic transitions at different times when Malthusian limits to growth are reached. If an aggregate of all these populations has a distribution that follows a power law, then all the larger exponential or power-law growth regions must have gone through a demographic transition *before* the global singularity is reached.

The term *demographic transition* is often used to refer to transformations at the regional or country level from having high birth and death rates to those of the modern era, involving low birth and death rates (Caldwell and Caldwell 1993). A *premodern demographic transition*, in contrast, is typically one from an exponential or power-law growth trend to a decline caused by high death rates. Exponential growth, of course, is simply what occurs when the growth rate definition for a single time period, $(dN/dt)_t = a_t N$, is extended to one in which the coefficient a_t for the rate of population increase at time t is relatively constant around a mean of a_0 so that $a_t \sim a_0$ is a good approximation over successive time intervals $t = 1, 2, \ldots, k$, resulting in equation (1).

Figure 1 gives an example of premodern population estimates for Hawaii up to 1800 (Dye and Komori 1992). It shows a quadratic growth trend followed by a demographic transition at 1480. Premodern demographic declines are usually caused by higher death rates resulting from a variety of mechanisms, such as emigration, mortality, fertility and child survival; these changes often involve Malthusian limits to growth, or declines associated with warfare, internal sociopolitical violence, emigration, and disease. Demographic transitions of the modern variety, however, are also usually preceded by periods of power-law (often quadratic) growth.

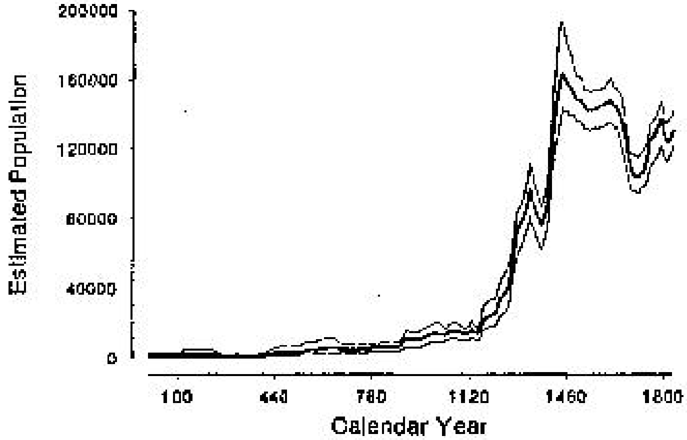


Figure 1. Exponential Growth Pattern for Hawaiian Population Estimates, 1 – 1832 CE, with 95% confidence intervals (Komori 1992)

It is 45 years since von Foerster, Mora, and Amiot published their article in *Science* predicting from the power-law trend in world population a world demographic transition that would occur well before 2027 (the estimated temporal singularity of the trend). The beginning of that transition did in fact occur with a shift in the growth curve after 1962. Further, that shift, evident from 1962 to 2004, vastly exceeded the amplitude of previous cyclical deviations from the trend, as we show below. The power-law trend in world population growth that obtains prior to 1963 runs for hundreds if not thousands of years.

The von Foerster prediction and explanation of population phase shift is not invalidated by the fact that power-law growth at the world level does not entail power-law growth at regional or country levels. As noted, such demographic transitions at the world level tend to occur only after most of the regions or countries having the largest populations have gone through country-level transitions. Nonetheless, the basis of their prediction and its significance has been largely forgotten (but see Umpleby 1990) by demographers now busy charting the mechanisms of changing growth rates and whether the new trends are toward near-zero growth. Predictions of transitions close to the singularity point implied by the slope of any observed power-law growth trend—in this case, the von Foerster "doomsday" of 2027, well before which a world demographic transition *had* to occur—are predictions not about mechanisms but explanations that derive from the power-law singularity implicit in a power law for world population growth. It predicts and explains a world demographic transition as the calendar date approaches the date of the singularity. The validity of the prediction does not imply any particular mechanisms, however, by which this shift will occur, nor does it entail that regional growth curves will go through a demographic transitions at a uniform time. Most European countries, for example, went through demographic transitions between 1870 and 1930.

China is the single largest population for 1 to 2000 CE that is aggregated into world population growth figures, and its growth is best fit by a power-law growth curve. China's overall population growth from 0 – ca. 2000 is shown in figure 3 (Heilig 1999). Even if China had an exponential growth curve, however, the world growth curve might still follow a power law.

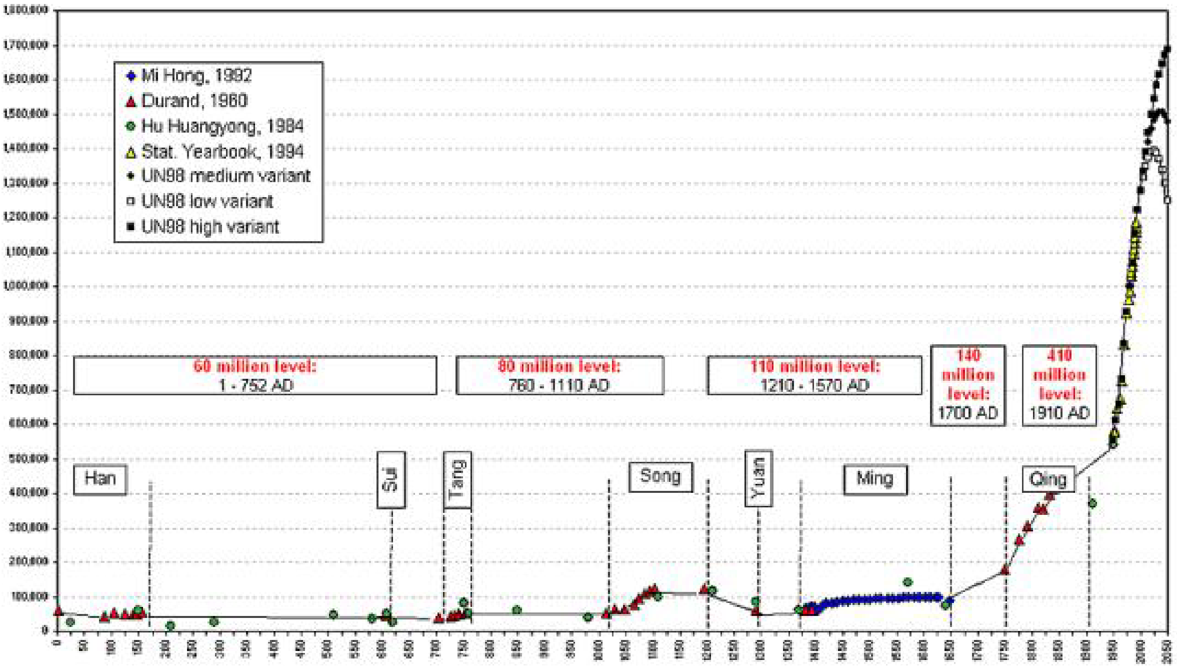
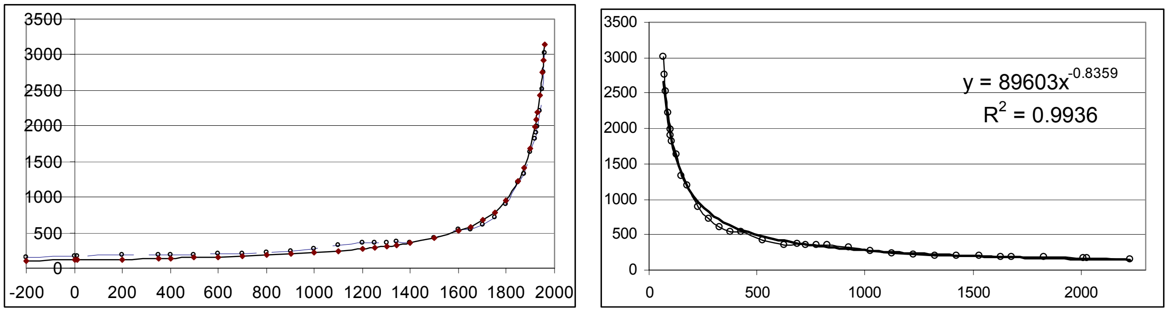


Figure 3. Exponential Growth of Chinese Population in Millions, 1 – 2000 CE and projections to 2050 (*Population. History and Projection, International Institute for Applied Systems Analysis*)

Although a world population growth curve that is power-law in shape does not always reflect regional growth curves, it is important to recognize that accurate estimation of these world-level curves, when they apply, is useful for other reasons. There is a major difference between identifying population growth mechanisms and the type of prediction made by von Foerster by estimating singularity dates for a power-law trend. The von Foerster prediction used twenty relatively accurate population estimates between 1975 and 1958 that fit a power-law growth trend with an R^2 of 0.9957 for a slope that is also consistent with population estimates at the start of the Christian era. Considerable work has been done since 1960 (for example, by Kremer 1993, Chandler 1997, and Modelski 2003) to arrive at reliable world population figures for dates prior to 1750, so that we may meaningfully ask whether this new evidence is consistent with a continuous 2,000 year power-law growth trend. We report our findings on this question here, using Kremer's and other more recent estimates of world population growth, but we also ask: (1) How many shifts are there likely to have been in world population growth trends? (2) Have there been any earlier periods in which human population growth was so rapid that a singularity would have been encountered in some earlier historical period, and explaining the occurrence of an earlier world demographic transition? (3) If there have been prior demographic transitions that were not due to an approaching population growth singularity, what were the likely mechanisms involved?

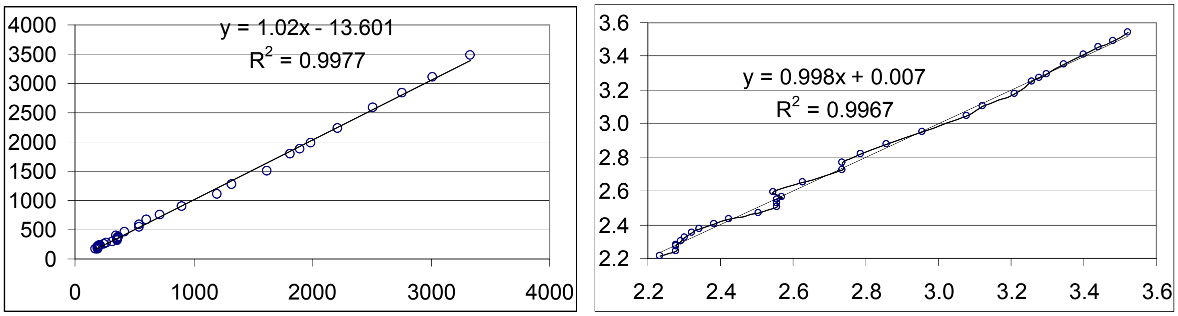
Cohen (1995:341) found faster-than-exponential world population growth from 1400-1970. We find, however, that von Foerster's trend goes back to 200 BCE, using reliable evidence over 35 world population data points from Kremer (1993) supplemented with estimates from the

World Bank (2004) for recent years, along with the exact same methods used by von Foerster, His data and ours correlate very reliably ($R^2 > 0.999$). We get a slightly better fit than he did ($R^2 = 0.9977$, figure 4) to a power-law curve with essentially the same slope, each within a standard error of the other. Thus, we confirm and extend von Foerster's finding, and converge on basically the same singularity and the same predictive explanation for the world demographic transition that began in 1962. Contrary to Cohen's (1995) assertion for fit of the period 200 BCE to 1400 to exponential growth, the $R^2 = .9223$, inferior to fit of the longer power-law trend. Note that the power law in figure 4b is not quadratic as in 2b ($y = K/x^2$) but subquadratic ($y = K/x^{.8359}$), and the fractional reductions are not 1/4th, 1/9th, and so forth, but approximately 1/4th, 1/6th, etcetera. Johansen and Sornette (2001: 19) get results very similar to ours from UNPD data starting with 0 CE (Figure 12) with singularity in their model—which also takes cycles into account—occurring in 2050.



(a) Double fit least squares $K=1.88 \times 10^{11}$ and $k=0.973$; $R^2 = .994$ log-predicted/log-actual

(b) Right: Excel Power-law fit, $y = (8.96 \times 10^{11})^{-.8359}$ $R^2 = .9936$ predicted/actual



(c) Correlation between trend line/actual data *(d) log-log plot for figure (a)*

Figure 4. Fit of Actual (Y axis) World Population in Millions for 200 BCE – 1965 to Power-Law Prediction (X axis), and log-log fit

Estimating a power-law fit of world population growth during particular historical periods is useful for another reason beside that of predicting singularities: Namely, to compare the prediction line for constant power-law growth with the actual population figures so as to identify detrended population cycles. Because we have 15 time points between 200 BCE and 1750 in our data series, rather than the two of von Foerster, we can contribute to the study of long-term world population cycles. The detrended population change estimates shown in figures 5 and 6 also give a much more accurate account of world population cycles than ones improperly detrended according to exponential growth. As such, they may provide some useful variables for dynamical analysis of how population trends averaged over many regions may interact with other variables, both in the aggregate and at regional levels.

Figure 5 shows our detrended cycles of world population growth, subtracting the von Foerster/White power-law growth trend. The cycles are decreasing in length but of increasing amplitude. Roughly 3.8 such cycles occur before the power-law trend ceases to operate, and growth rates start to plummet after 1960 after a brief up tick. Figure 6 shows the same cycles but normalized to scale as percentages of the sizes of populations in the trend line, so that the amplitudes of deviation are comparable and somewhat more even.

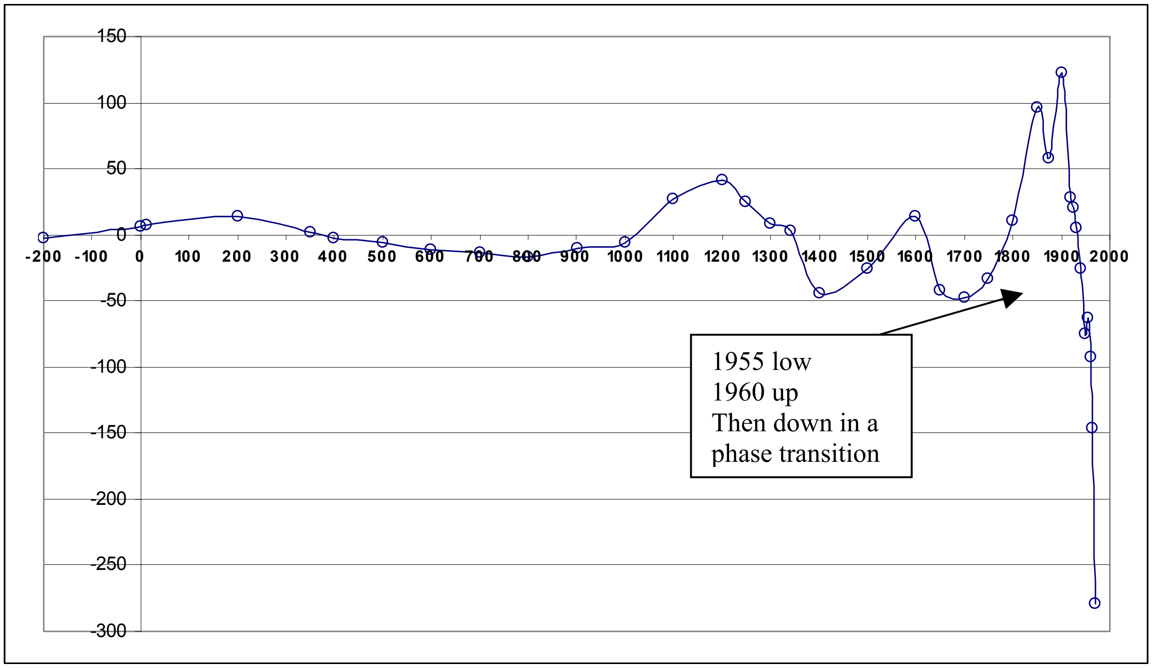


Figure 5. Cycles of population deviation from the power-law slope of 200 BCE to 1962

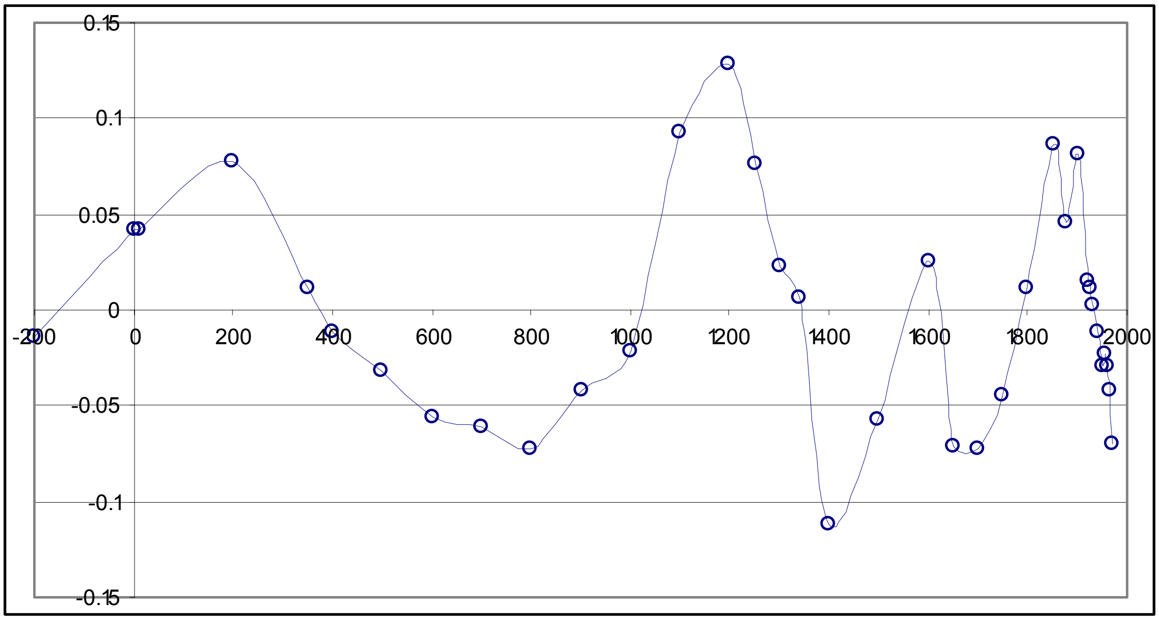


Figure 6. Cycles of percentage deviation from the power-law slope of 200 BCE to 1962

These population cycles are familiar to historical demographers and sociologists (Goldstone 1991, Fischer 1996, Wilson and Airey 1999, Nefedov 2001, Turchin 2005). These oscillations might be shortening in connection with the log of time to singularity (see Converging Results). Alternately, because the ramps up to the peaks tend to correspond to warming periods (Galloway

1986: Figure 7), they might be climatically induced. For England and China, in addition, Turchin (2005) finds evidence of a dynamic interaction between the peaking of population cycles and the somewhat lagged peaking of sociopolitical violence. Generalizing this hypothesis to a world scale, we test this relationship by measuring the correlation between these cycles and the cycles of consolidation and fragmentation in percentages of world political territories held by different polities, as coded and normalized—to number of effective polities (y axis)—by Taagepera (1997). The dotted lines in figure 7 are the highs and lows of the cycles correspond to those in figure 6. Taagepera's data is summarized in figure 7. The correlation is very high, as shown in figure 8. Turchin's hypothesis explains the correlation where a black dot (labeled Taag) is located above a white dot (above in time on the Y axis) for congruent population cycles. From 1250 onward, however, the two transitions occur more often synchronously or in reverse order to what is predicted by Turchin, although his hypothesis concerns country-level dynamics.

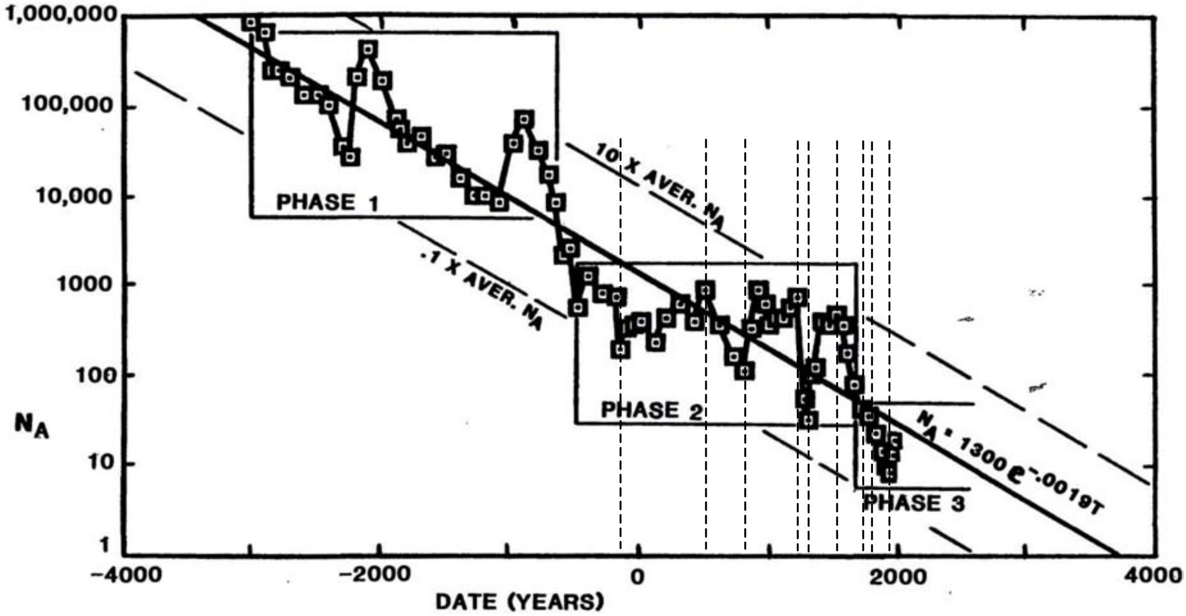


Figure 7. Cycles of Political Consolidation (lower values on Y axis) vs. Fragmentation (*Taagepera 1997*)

Figure 8. Congruence in the Phasing of Political (Taag) and Population cycles, Y axis = Historical dates, X axis = alternate high/low

Convergent Results

Five convergent validations support our findings of trends and cycles in the period 200 BCE to 1962. One is that we validate the population trends and cycles in figures 5 and 6 against those of Taagepera (1997) for political fragmentation and consolidation. A second is that Johansen and Sornette (2001: figure 13) validate their findings on population trends and cycles—which are very close to our own—with data on the economic output of human beings over the last 2000 years (data on world GDP) and on market indices over the last 100 or more years. Third, their evidence that the singularities for all these different indices are the same—2052 ±10 years—and is not only identical with the world population growth singularity but close to the singularity estimated by von Foerster. Their singularity date is slightly different that ours and von Foerster's, but they are using a more complex model with parameters that take into account both trends and cycles. Fourth, their results on cycle lengths and oscillations converge with ours—arrived at totally independently—in that the length of cycles in the last 2000+ years is shrinking with the log time to singularity (log-periodic shortening of oscillation; Canessa 2001). Fifth, while Johansen and Sornette use a data series that only goes back to 0 CE, our analysis of period breaks in the longer data series provided by Kremer, our finding of a power-law fit back to 200 BCE converges with Kremer’s (1993: p 703, tables II-V) report of a break at 200 BCE in the time-series trend and his choice of 200 BCE-present to model population trends. Kremer, however, attaches no particular importance to a breakpoint at 200 BCE and does not try to periodize population transitions earlier than the one that is operating in the present

Public Knowledge and Policy Issues

Curiously, von Foerster et al.’s (1960) finding of a population growth singularity in the present era, and its implications, has never found its way into the popular press. Wilmoth and Ball (1992), in reviewing the population debate in the U.S. in popular magazines in the post WW II period, for example, find no mention of power-law growth, singularities, or the instabilities associated with them. Greenhalgh (2003), in her demonstration of how demographic scientism was adopted for purposes of political agendas of state control in China, finds no evidence of any impact or mobilization of the power-law growth, singularities, or the instabilities paradigm. Rather, she argues, it was the Club of Rome limits to growth models of Meadows et al. (1974) and Mesarovic and Pestel (1974) that were mobilized in this political-scientistic movement (one of the odd thing about the Meadows et al. book, in fact, was that it referred to world population growth as exponential rather than hyperbolic or power law).

Significantly for macromodeling, the multivariate modeling of Johansen and Sornette (2001) shows, “in theoretical support, … coupling of population, capital, R&D and technology *though the isolated dynamics do not exhibit it*” (italics ours to emphasize the macro rather than micro dynamics). For the cross-over period prior to the singularities encountered in their, our, and von Foerster’s predictive explanations from power-law growth, they also examine scenarios for the new regime that we are well into today. Von Foerster et al., however, never developed the implication that a transition at singularity is not simply an “event” that begins as a certain predictable date, but involves cascades of singularities and instabilities that occur at different scales (Johansen and Sornette 2001:7-9).

The policy relevance of population singularity predictions, then, is their connection at the macro level to singularities in other areas such as economic instabilities, market crashes, and, potentially, political system crashes. The viewpoint common among demographers, that demographic transitions are strictly a matter of understanding micro-behavior at the individual level, is untenable.

Problems of Estimation and Exogenous Models

Given that population singularity transitions have acquired a new significance in the last few years, arising in the complexity sciences research community, it is crucial to revisit the issues of estimation methods. This we do in a separate article (White, Malkov and Korotayev 2005). To summarize those findings, there is not a unique solution to the problem of establishing fit of an empirical population growth curve to the four parameters required in estimating power-law trends (but see below under K and the Hyperbolic). Rather the estimation problem is one of assessing multiple locally optimal solutions to parameter configurations that we show can be quite different. These local solutions are differential sensitive to the nonrandom cyclic deviations from the trend line. Hence it is necessary to use estimation methods such as simulated annealing that utilize different initial choice of parameters to find local optima, and generate a range of solutions. Johansen and Sornette incorporate cyclic deviations from power-law trends into their estimation procedures, introducing an additional set of parameters. Again, this does not lead to unique solutions to the problem of best-fit estimation. Their method recognizes the problem, however, and they use simulated annealing to find a variety of different solutions to parameter estimation, and estimate the range of error not from a single model from the range of variation in locally optimal models. All of the models for world population trends to date, however, show a population singularity in the range of 2040 ± 30 years. A singularity transition, however, is not a single event but cascades of singularities and instabilities both in demographic processes at different scales in economic, political, cultural and religious phenomena. A qualitative assessment of the period in which such instabilities would unwind in a major world transition might occur over time scales up to a century in length, operative at many different smaller time and regional scales. Because of the uniqueness of the world population singularity, however, this century of instability will undoubtedly have a great many unique properties, both positive and negative, as a transition of this sort has never been witnessed in human evolution. Kapitza (1996) takes a different view than Johansen and Sornette, that in "the modern interconnected and interdependent world this transition will practically come to an end within the next 50 years. It is happening much faster than in Europe *where a similar process began at the end of the XVIII century*" (italics ours to emphasize his sense of lack of uniqueness of singularity transitions). In contrast to Johansen and Sornette (2001), Kapitza imagines the singularity as a demographic transition to a stable population of some 14 billion, "determined by the inherent pattern of growth of an open system, rather than by the lack of resources."

Remaining Questions

To return now to questions posed earlier, we can not provide some answers:

(1) How many shifts are there likely to have been in world population growth trends?

-- We find evidence for one shift in -5000 from exponential to high-slope power-law growth, and another in 200 BCE from high- to the low-slope von Foerster power-law growth.

(2) Have there been any earlier periods in which human population growth was so rapid that a singularity would have been encountered in some earlier historical period, explaining the occurrence of an early historical demographic transition?

-- No. The world demographic transition circa 200 BCE was not predicted by an approaching singularity.

(3) If there have been prior demographic transitions that were not due to an approaching power-law population growth singularity, what were the likely mechanisms involved?

-- The mechanisms involved circa 200 BCE in the world demographic transition were most likely: (1) innovations in the organization of the state, the state bureaucracy, and the rapidity with which information could be transmitted over long distances by the use of couriers posted at successive sites traveled by horseback (Taagepera 1997:486); and (2) the invention of philosophies and belief systems that were also transmitted over larger regions of civilization, including the diffusion of the ancient world religions, of which Buddhism, Hinduism and Christianity were examples.

Explanations and Mechanisms

From paleoontological, archaeological and historical evidence, we characterize in table 2 what we know about the four periods and of differing world population trends the three world demographic transitions between them. We distinguish sharply between explanations and mechanisms. A population growth singularity that predicts a world demographic transition is an explanation because the transition is unavoidable, but the explanation does not entail any one or particular set of mechanisms, only that the transition will occur. Under *Transition explanation* we list those that apply to world rather than country level modern or premodern demographic transitions. The mechanisms involved in those transitions, which must of course contribute to world demographic transitions, are noted under *Transition mechanisms*.

One of the potential disturbing things about the phases and transitions noted in table 2 is the fact that the data that we use to make these assessments, those of Kremer, come from four different sources, and the sources themselves are correlated with the periodization, which is a possible source of artifacts in our results. The period from 5000 to 200 BCE, for example, is from archaeological estimates. These will need to be redone from new compilations of recent estimates of early city size growth (Chandler 1997, Modelski 2003) and from new archaeological findings. While there is little doubt in our mind that our qualitative results are correct as to periodization of different trends and transitions, more careful data estimation needs to be done for this period in particular.

World Period	World Trend*	Connectivity	Generative Process	Geo-Centroid	Transition Explanation	Transition Mechanisms
From 1962/63	Decline**	High	**	India	n.a.	n.a.
-200 to1962	Power-law	Low	World-Systems	China	Mathematical Singularity	e.g., Cost of Children
-5000 to -200	Power-law	High	Radial Diffusion Colonizing Cities	Near East	Intercity trade network	e.g., Political Organization
1M to -5000	Exponential	None	Segmentation	Africa-Old-New World	Filling of Earth Surface	Nucleation of Populations

* These trends begin often earlier when examined region by region and power-law distributions do not usually carry over to regions.
** Whether the decline is exponential or power-law is not yet evident, nor can we yet confidently describe the type of period.

Table 2: Summary of Periods and Trends, with Explanations and Mechanisms for World Demographic Transitions at the end of a world period trend

Kremer (1993), Cohen (1995), and others have constructed models in which population growth drives technological change to allow further growth. The more recent, by Cohen (1995:343-344), is a variant of the logistic equation that gives a period of growth that is exponential, followed by one that is faster than exponential, finally giving way to sigmoidal growth towards a finite limit. This would be particularly apt for our period 200 BCE to 1962, were it not for the fact that this period is preceded not by exponential growth but steeper power-law growth that the period which follows. Cohen treats our population cycles in Figure 7, up to 1400, as a "period of exponential growth," ignoring the evident cycles, and ignores population estimates prior to the Christian Era (0 CE).

Some interesting temporal nonlinearities are evident in table 2. Connectivity (column 3) does not increase linearly with time. The generative processes associated with different trends are qualitatively different. The location of the geocentroid of maximal growth does not move in a linear direction. Three different explanations are required for world population transitions. Mechanisms in the last column could be amplified along different dimensions which might exhibit complex nonlinearities through time.

We are currently living in the first era of human evolution in which the major transitions underway are associated with a massive growth singularity. Johansen and Sornette (2001:7-9) show convincingly that the transitions that occur near growth singularities are not isolated events but that cascades of singularities and instabilities occur at different scales, and at each of these time scales—taking the behavior of dozens of stock markets over hundreds of years in their demonstration—the signature of a locally impending crash is a log-periodic shortening of cyclic fluctuation such as we observe also in population behavior at the world level. Whether such fluctuations are observed prior to regional population declines needs further research.

In terms of these limited and provisional findings, there is a simple theory of the four phases of world population transitions from 1 MYBP. Up to 5000 BCE communities that grew to a certain size tended to segment and hive off into segments, usually two splitting from one. Some communities might die out or if they grew too small merge back into others. The rates of splitting and merging would have varied around a constant, with doubling time for the world population occurring in equal intervals. This would predict aggregate population growth that is exponential.

Once space for further segmentation was filled, further world population growth could only occur if some means obtained for either densifying migratory population in the same area, which could happen with domestication of animals, or organizational nucleation that extended the limits of community size and eventually, town and early city size. Early towns and cities, however, were likely to be interdependent within networks of exchange, development of new forms of trade, and, eventually, colonization of new cities by hiving off and resettling further out within a growing, interdependent, interurban network. If the population of each settlement grows exponentially but with different limits to growth set by position in the interurban network as well as environmental constraints, world population would shift to power-law growth simply as a function of the aggregation of differential exponential growth at local levels. If power-law growth continued at a constant rate, it would eventually encounter the type of singularity that we witnessed in 1962/63. The explanations for two shifts among three phases would be basically mechanical. One additional phase shift occurred, however, in about 200 BCE which must have represented a true organizational innovation: the invention of faster mechanisms for the transmission of messages and the spread of information, e.g., the relay systems of empires. Innovations of this sort would be cumulative, in that the technology for rapid relay and road transport would vastly outlive the time span of empires and their collapse into state systems. Curiously, this organizational innovation seems to have been accompanied by a slowing of the power-law rate of growth. Presumably, while the earlier system was a convective, point to point process of colonization that created and expanded interurban trade networks, the new organization system capable of faster transit over the entire network would create a system where information would spread not convectively but through wave propagation, spreading the energy previously concentrated in convective links. Because entropy cannot decrease in the reorganization of energies in a system, we might postulate some mechanism whereby the power-law rate of world population growth would actually have to decrease to slower rates to conserve thermodynamic equilibrium. That transformation, not yet well understood as a human historical phase transition, might well be worthy of further research.

Macro and formal demography even at the most general level, that of world population dynamics, can pose questions, give explanations, and require us to examine more closely the links between macro and micro as they affect demographic transitions. Our results are a case-in-point that supplements those discussed by Lee (2001) for the use of formal demography to identify micro-macro linkages. Formal demography—in this case, macro modeling of the types and implications of world population transitions—proves to be a field that can give us theoretical explanations. Our macromodeling of world population focus on where and how to study the actual mechanisms that connect the macro-level phenomena in the phasing of cycles in figures 7 and 8 and the variables of connectivity, generative process, and world system geographic centroid in table 2.

Kapitza's Synthetic Model

Kapitza (1992, 1993, 1996), with a different set of goals than ours, and using a synthetic systems phenomenology derived from physics rather than a more analytic approach, arrives at an expectably different set of results than our four phases in table 2. He identifies a large-scale parameter Ḱ as describing "all relevant proportions in the phenomenology and interprets Ḱ as "the natural size of a coherent population unit" (1996:5.5) with only three dynamic transitions: (A) from 4.4 to 2.8 MYBP (million years before present) with oscillation around a coherent size relative to Ḱ, (B) the epoch of power-law (hyperbolic) growth, which he sees as unitary from 2.8 million years (toolmaking) to the 20th century. What we regard as three very different phases (and perhaps more prior to 1 MYBP) he regards as one, but his data confirm ours in having huge deviations from his continuous model for the period 5,000 BCE to 0 CE. His third period is (C), the contemporary demographic transition: "This rapid non-equilibrium transition leads to the break-up and disruption of traditions and customs long established in human societies, factors that to a large extent stabilised the life-style in the past, setting up long-term correlations between generations. Today it is customary to say that the connections between generations are severed and many see in this one of the reasons for the strife and stress of modern life, a factor that should be attributed to the transition through which we are passing" (6.1). It "displays many features of a phase transition." However, "It is the age distribution of the population that changes during the transition and this is the most important thing happening" (6.2). Like Johansen and Sornette, however, he finds a log-periodic shortening of the phases of transformation ("cycles" with increasingly foreshortened length) in large-scale historical events (7.1). After the transition that he sees occurring today (1996), however, he envisions a massive reorganization time that is not fully discussed. Further, he applies the standard Lyapunoff criteria of effects of initial conditions to his equations of population dynamics to identify the time of "maximum chaotic instabilities" around 1965 but sees a change in sign before the singularity date is encountered that signals the possibility for systemically stable development. He warns of the potentially destabilizing effects of conflicts. He sees in fractal patterns in spatial distributions, city-size hierarchies and the power-law distribution of wealth, however, an implication that: "In the global population systems a number of instabilities may develop and various stabilizing factors," including migration. "This is best demonstrated by the large scale cycles observed in the human population system, *although a whole hierarchy of instabilities of a lesser scale do develop*" (8.2). This marks a return to the theme of Johansen and Sornette, except that for Kapitza instabilities are not predicted at all scales but only at lesser scales. The agreement among these two sets of authors and certain of the historical demographers who do dynamic modeling, such as Le Bras (1992), is that it is much more important to understand the internal dynamic of the systemic interactions in phase transitions than to argue from environmental limits, as did the Club of Rome.

Endogenous and Exogenous Models

Endogenous population models of the form $dN/dt = f(N, a_0)$, if a_0 is an estimated coefficient are trivial because they are simply an extrapolation of a trend and a_0 is not explained. Exogenous variables are required. Johansen and Sornette (2001) and Kremer (1993) add exogenous economic variables to their population growth models, although Kremer does so by considerable use of proxies to unmeasured variables.

The mistake of contemporary demographers is to look for those variables exclusively at the micro level. The mistake of many analytic modelers is either not to explain estimated constants or to posit exogenous variables that are immeasurable, which may amount to the same thing analytically. Kapitza's model, however, is entirely endogenous, and some of his formulations offer additional insights and unexpected systemic properties that strengthen our ability to estimate models empirically and thus to obtain testable predictions (see below).

Endogenous trend models of the power-law form, $dN/dt = f(N^r, t_o, k, K)$, are trivial up to the point of predicting instabilities due to singularity t_o, but r, k and K are not explained even if t_o can be estimated within broad error limits. Again, exogenous variables are required. Deviations from endogenous trend lines in these cases become the cycles of various lengths—predicted by the model to decrease in length in proportion to closeness to singularity—that require explanation by exogenous variables. Hence there are two sources of useful complementarity between endogenous power-law models and exogenous models, provided the latter utilize measurable variables.

What is not commonly understood in identifying explanatory variables for population cycles, however, is that simple correlations or time-lagged correlations in themselves are not adequate explanations of dynamics. What is required for dynamic explanations of cycles, where $\Delta_1 = dN'/dt$ is change relative to a trend line, is not a correlation between Δ_1 and some changes variables $\Delta_2, \Delta_3, .., \Delta_m$, but a function with state variables, $\Delta_1 = f(x_1, x_2,\ldots, x_i,\ldots, x_m)$, where for each x_i there is also some $\Delta_i = f(x_1, x_2,\ldots, N',\ldots, x_m)$, such that there is dynamic interaction between the variables, which can only occur if there are reciprocal time lags, i.e., dynamic feedback relationships between them. Turchin and Korotayev (2005) and Turchin (2005), for example, show that detrended population cycles interact dynamically with off-phase cycles of sociopolitical violence, and Turchin (2003) explains the theory behind off-phase dynamics, the idea being that the phase diagram of dynamically interacting variables should not be a straight line but have some open curvature and circular closure to be predictive. The involvement of sociopolitical violence in population cycles suggests that Kapitza's view of only lesser scale or minor instabilities in the current singularity may be mistaken, and that the cascading instabilities at different scales of Johansen and Sornette (2001) needs more careful investigation.

Kapitza's view that the most important variable affecting the dynamics of a post-transition world will be the major upward-age skewing of age distribution, and the implications for intergenerational stability (Brudner 1986) bears closer examination.

Lending Credence to Kapitza: K and the Hyperbolic

Comparing Kapitza's synthetic modeling to the analytic modeling of von Foerster and others, one of his most interesting assertions (1996:5.5) is that the constant $\acute{K}$ (related to K) "can be interpreted as the natural size of a coherent population unit," accompanied by way of demonstration with the formula $dN/dt= N^2/\acute{K}^2$ for his period B where growth is hyperbolic. As a simple test of this hypothesis, figure 8 plots N^2 in millions squared on the Y axis of figure 8 and dN/dt in millions/year on the x axis. Then we estimated the slope $K = (N^2 /(dN/dt))^{1/2}$, dimensioned in millions for the dotted line, which takes follows the population curve up to 1970. By this calculation, $\acute{K}^2$ = 200 billion people $\approx K = 1.88 \times 10^{11}$, from Table 4(a), which is four doublings of the existing world population. It would appear that (Kapitza's) Ḱ = $K^{1/2}$ (von Foerster's), and that Ḱ = 447,000, which gives an estimate of about a half a million as a "natural size of a coherent population unit," which might begin to make sense in terms of urban centers. What is particularly interesting about the curve in figure 8 is that with the singularity predicting deflection upwards and away from a straight line for growth rate proportional to the quadratic N^2, the actual curve follows a true hyperbolic. This is the first empirical evidence of an empirically estimated hyperbolic growth law deserving of the name.

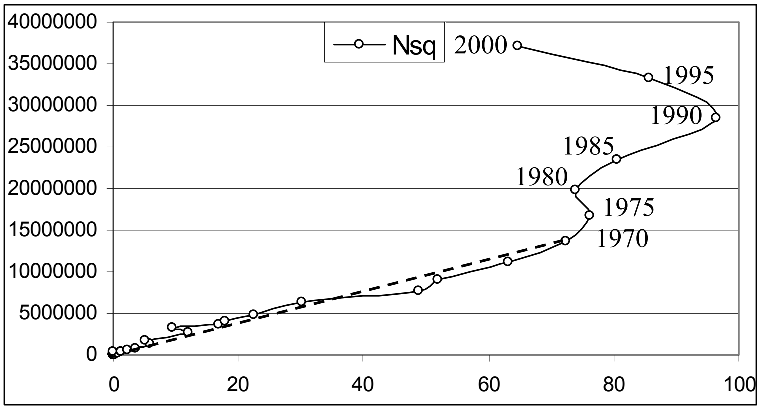


Figure 8: Test of Kapitza's hypothesis, plotting x = dN/dt, and y = N^2

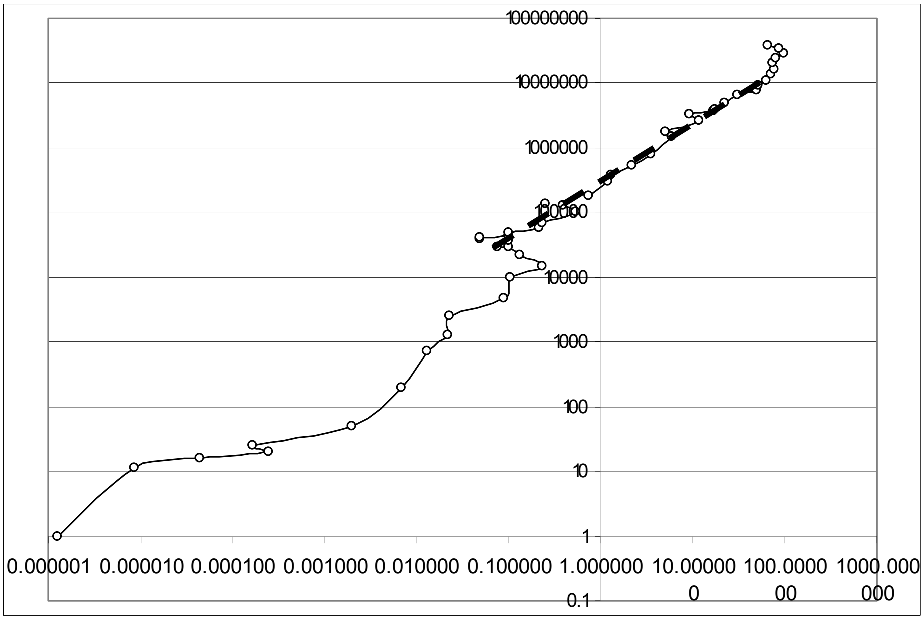


Figure 9: Kapitza trend line, plotting x = log (dN/dt) and y = log (N^2)
(trend line is for 200 BCE to 1970)

The direct computation of K in this manner is more reliable than discussed for von Foerster's method because one point in the regression line (figure 8) is rooted in the origin and there are no other parameters to estimate. Because the datapoints in figure 8 are bunched around zero, it is better to do this regression on a log-log graph, as shown in figure 9. This reduces the uncertainly in estimated the constants in the von Foerster method from four (k, K, starting date, singularity date) to three (k, starting date, singularity date). It also assures fewer locally maximum solutions in the regression that estimates the power law.

Conclusions

The empirical study and theory of world population dynamics is in its infancy when considered in terms of reliable measurement of relevant variables, including population, understanding the analytic approaches that allow certain types of dynamic predictions, developing both grounded analytical and synthetic models, understanding how to arrive at valid macrodynamic explanations, and connecting macro to micro processes. The study of world population growth is dependent on archaeological and historical estimates that have only recently become available in reliable form. The analytic methods introduced by von Foerster, Mora, and Amiot (1960) allowed certain dynamic predictions to be made about singularity transitions and has opened new fields of study, but has made almost no impact on public or policy considerations. Until methods and findings stabilize it may be too early to push for such impact of mathematical modeling. We have used the insights of the exponential and von Foerster types of endogenous population models (or others in table 1) to distinguish world demographic transitions from premodern and modern country level demographic transitions and to set ourselves an analytic task of distinguishing empirical trends in world population and the transitions between them. We are not the only ones to begin to explore the interrelations of population trends and transitions with forms of world-system interconnections from 5000 BCE forward, but we have found a new and interesting interconnection between population transitions and political transitions as studied by Taagepera (1997).

Among both the synthetic and analytic modelers of world population growth, there is almost complete agreement that there is a unique singularity involved in changes human society is currently undergoing. Different studies have implicated population, economics, politics, wealth and age distributions, and other factors—including sociopolitical violence—as mutually interacting variables at the macro level. On the issue of population cycles, the authors of all the studies reviewed (chosen because of their analytic focus or synthetic relevance) are also in agreement that these and certain of the related cycles—and perhaps the dynamics of their interaction—tend to conform in actuality to log-periodic shortening.

Perhaps what is needed in these studies at the current time is the development of better measures and datasets for the testing of hypotheses, coupled with better understanding and use of analytic tools. Possibly the least profitable approach is to claim explanatory status for further mathematical models where most of the variables remain unmeasured (there are many ways to fit curves with many free parameters to an empirical times series). The fact that a useful cross-fertilization of analytical and synthetic modeling has emerged in this field might instead provide a basis for identifying new substantive and measurable variables for the testing of hypotheses about world system dynamics related to population. Clearly, any such models as are to be tested will gain from regional replications, where regions may also differ in a variety of measurable ways.

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Additional Abstract:

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