v1.1.4 Generative modeling of city-size scaling laws, 250 BCE – 2005: Embedded co-evolution in the theory of long-term geopolitical dynamics *

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Abstract

Scaling world city size distributions are analyzed in terms of q-exponentials, the distributions which naturally emerge within nonextensive statistical mechanics. These distributions allow to estimate hierarchy and scale parameters for 29 historical periods, thus conveniently replacing the Zipfian scaling law by an historically sensitive and nuanced model of how long-term oscillations in urban hierarchies co-evolve with urban industrial, commercial and financial innovations. Our present model emerges from attempts to understand our scaling results by validation tests using multiple political and economic variables. We investigate the model so as to explore the extent and by-play of endogenous dynamics and exogenous shocks as causes of synchronization. One such step consists of identifying a network macro-structural variable, the average urban hub adjacency ratio, to help explain interactive network dynamics of endogenous synchronization. We find, further, in examining our validation-check variables, that innovation clusters in space and time – the leading sectors of urban economies – are differentially embedded in periods of alternating hierarchical and heterarchical urban scaling regimes. Urban infrastructure and demography, as expected, change slowly compared to the pace of Schumpeterian K-cyclic tendencies in bursts of competitive economic innovation, but the synchrony of these changes in time and space, while a surprising discovery, is consistent with our network-economic theory. **[DOUG: I do not understand the sentence above.]** Sandwiched in an intermediate layer of change in terms of its spatiotemporal scalings we find the shifts in the leading cities and states, as studied by Modelski and Thompson, forming a synchronously-embedded three-layer model of economic, political, and urban structural oscillations. These findings have potential for unifying a new perspective on glacially slow and then quickly triggered structural changes in the city hierarchies and infrastructure with perspectives on long-cycles as studied through the world political and economic perspectives.

1 Introduction

We examine all of the most populated world cities in each of 29 historical periods and model the full spectrum of the size distributions in each period by independently fitting a mathematical function that is motivated

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by its connections to generative processes. Because we use world city data for each period (we use the entire population in largest cities for each period), our fitted parameters are not affected by sampling problems. Our choice of function incorporates the intuition that city growth is more closely coupled in the upper part of the size distribution to rapid political or economic changes, which only asymptotically approach power-law. Prominent cities might attract or repel inhabitants, for example, not at the same rate as smaller settlements, but at a rate that is more proportional to their size. As known from the study of city rank-size (Auerbach 1913, Zipf 1949) and Pareto (1897) distributions, we do not expect a uniform power-law slope over a whole spectrum of cities. We consider q-exponential functions that are not power-law in the lower size range but approach asymptotically a power-law slopes in the upper range and even allow the possibility of two distinct power-law slopes in the upper range [**Note: While we do not explicitly fit the 2-slope model it is mentioned later on when examining our curves.**] Given that our choice of function allows close fit to data on distinct historical periods, we address whether there is invariance over time in the fitted parameter that expresses the shape of world city-size distributions. We find instead that there is considerable variation in the shape parameter (analogous to the slope coeffificent of inequality for a Pareto power law). Because this variation is temporally autocorrelated, we examine whether the variations over time show gradual oscillations or sharp breaks between relatively stable periods. As there is a preponderance of discrete historical periodization, we examine whether there are clear historical correlates associated with these periods. We look to evaluate and do find strong evidence of validation for our interpretation of the generative changes that are likely to affect the fitted shape parameter for the function we employ in the model.

Rates of change proportional to size are likely to reflect processes that generate power laws. Attraction to cities indexed by employment numbers in 17 functionally skilled categories (e.g., construction, trade, education), for example, are found by Lobos (2005) to scale by power laws with year 2000 city size in U.S. metropolitan areas. As Krugman (1996a,b,c) notes, however, there has been no agreed-upon explanation for the rank-size law of cities. Various models deal with hierarchical organization in the fractal geometry of area distributions around cities (Batty and Longley 1994), how cities partition the plane (Buldyrev et al. 2004, Malescio, Dokholyan, Buldyrev and Stanley 2000), how transport corridors are organized (Carvalho, Iida and Penn 2003), and how city perimeters and area distribution of systems of cities affect city morphology (Makse, Havlin and Stanley 1995).

Physicists Marsili and Zhang (2004) are among those who have looked at city size as a function of interacting individuals and features that organize interaction. They start from a model of a reservoir of population with a fixed transition probability to becoming a city dweller and a small fixed probability that a new city will be formed. They model the case in which city growth is linear rather than power law (superlinear). They then introduce a variant assumption (Zanette and Manrubia 1997) that processes governing urban growth are not independent, and show that modeling the influence of individuals' decisions on one another through network interaction generates a Zipfian effect. Transition-to-city rates for cities of size m now shift from linear in m to a mixture of linear and quadratic effects. This reflects a mix of individual decisions some of which are independent and some nonindependent due to network interaction. Their model posits a city transition size m_0 at which Zipf's law begins to hold: for a city with $m \ll m_0$, growth is constant while for $m \gg m_0$ growth is proportional to size, and follows a Zipfian. Their intuition for this model accords with Zipf (1949), for whom the scaling law is valid only for large cities. Our model accords with this observation but instead of a predetermined transition size the transition to power-law tails emerges from the function itself.

We present a model of city-size hierarchies that follows the approach of Malacarne, Mendes and Lenzi (2002), who use Mandelbrot's (1977) generalization of Zipf's distribution to deal with fractality by a measure of the extent to which the long tails of various distributions show anomalous decay. Our model is weakly generative in that the phenomenology of the function we employ makes implicit the upper-size transition asymptotically to a power-law. It is potentially generative in a stronger mathematical sense in that the shape parameter in some applications in which interactions are well understood can be derived from microdynamics. We think this will prove to be such a case. The scaling model deals with a broad class of hierarchically organized and self-scaling processes and satisfies our requirements for a suitably general function with which to model city-size distributions historically. It also subsumes the mathematical model of anomalous decay and fractality under a q-exponential distribution (Tsallis 1988, Boon and Tsallis 2005). White, Kejžar, Tsallis, Farmer and White (2006) show, in a general model of network agency and feedback interactions, that variations in their network process simulation parameters determine the value of the shape parameter, q, of our mathematical function, the same as used here.

Malacarne et al. use the method of generalized monolog plots based on the generalized q-logarithmic function $\ln_q$ to estimate the parameter q, which measures an asymptotic (negative) slope $\alpha = 1/(q-1)$, as the function approaches a power law for the larger city sizes. The larger q is the faster the growth becomes power law, and the fatter the tail the distribution, thus also more inegalitarian. In this sense the q-exponential function subsumes a Pareto distribution for the upper city sizes, which in the special case also corresponds to the Zipfian. The generalized logarithmic function $\ln_q$ is well suited for studying the distributions of city sizes because it is somewhere between and inclusive of the distributions typical of small towns (with more size-independent growth rates) and those of large cities (proportional growth). In the derivation of Malacarne et al., q represents the mix between growth processes that depend on interaction to generate power-law distributions, and those that give the appearance of not depending on interaction (as if decisions were made independently by each individual as in the Marsili and Zhang city growth model). What occurs in the phenomenology of the q-exponential $f_q(x) = e_q^x$ is that the smaller the argument x the more independent is $f_q(x)$ from q. This gives the "illusion" that there is probabilistic independence. This comes from the remarkable property that $e_q^x \sim 1 + x$ for $x \to 0$ for ALL values of q. [**I think from here till the end of paragraph, the sentences are not correct. Constantino, can you look at the explanation?**] This case effectively converges to the result in which $q = 1$, which would also hold for constant rates of growth or decline. **check:**Although the processes are independent in this case, the rate of growth for a city is not constant but exponential. For small values of x, the growth might be linear, but for larger x it goes to (a) exponential when $q = 1$, and (b) power law when q is larger. The case of $q = 1$ is equivalent to that of statistically independent processes that generate Boltzmann-Gibbs entropy distributions.

For a q-exponential PDF $f_q(x)$ in which $q > 1$, including the large arguments of x, we are in the domain of a mathematical model of generalized entropy designed for mixes of interactive processes and processes that appear as noninteractive or independent because of low values of the argument x. This is the domain of q-entropy (Tsallis 1988) in which Boltzmann-Gibbs entropy is the special case in which $q = 1$. The Zipfian distribution for large cities becomes a special case of $q > 1$ subsumed under a more general distribution for cities of any size. The case where $q = 1.5$ for a cumulative probability distribution function (CDF) of city sizes, for example, corresponds to Zipf's law or a Pareto inequality coefficient of 2, which we call *egalitarian.*[1] At the other extreme, as q increases, in generating this CDF, the tails of the city size distribution becomes thicker. We call this case a more *hierarchic* distribution. We refer to cities as *hubs* if they are in the upper power-law range (with thick or thin tails depending on q) of the city size distribution.

What value of q would express the gap between a roughly Zipfian distribution of cities in the upper size bins and a "primate" city (a dominating city in a country due to economic, political causes)? Or the gap between a low to mid-upper end of a q-exponential distribution which approximates a power-law in the mid-upper range of city sizes but shows a city-size threshold above which there is a transition to a second power-law regime in the larger-size cities? This latter type of distribution, as shown by Borges (2005), is not unusual for the distribution of personal income and for that of gross domestic product at the county or country level. He shows the applicability of a (q_1, q_2)-exponential model to these cases in which there is a finite value m in the upper range of the distribution where one power-law slope takes over from another. These are rich-get-richer models but with a discrete "knee" above which the distribution branches to the super-rich, governed by a different power-law slope. No one to date has applied (q_1, q_2)-exponential models

[1] As q dips below 1.5, and the power-law slope at the upper city sizes becomes steeper than Zipf, it might appear that the city hierarchy becomes even more egalitarian as the central hubs (large cities) become even fewer. What actually happens in the CDF as $q \to 1$ is that the theoretical variance becomes infinite, so that with a finite number of cities the likelihood of "extreme hubs" appearing at all becomes vanishingly small, as if all the arguments for empirical size x arguments were "small," and thus convergent to the case where $q = 1$ for an entire size spectrum.

to the full spectra of city-size distributions, and we shall see in our fitting of q-exponential models that there is indeed a need for distinct q slopes in the tails for certain historical city-size distributions.

As a test of the applicability of q-entropic properties of city sizes, Malacarne, Mendes and Lenzi (2002:2) show that both the U.S. and Brazilian city sizes, down to the smallest cities, show a linear fit in the cumulative population distribution for city-size rankings for which $q = 1.7$ and $R^2 = 0.99$ in both cases, and thus the asymptotic Pareto or CDF power-law slope $\alpha = 1/(q-1) = 1.41$ with only three deviant "primate" cities (New York, Rio, and São Paolo) and no deviant smaller cities.

2 Urban Population Data

Data on largest city sizes for each of 29 periods from 430 BCE to 1950 from Chandler and Fox (1974) were homogeneited and georeferenced by one of us.[2] More large cities appear over time in these data but some of the smaller city-size bins were dropped in Chandler's coding so as to keep relatively constant the number of largest cities coded in each time period. Rozenblat also provided U.N. data for the largest cities for each of 12 periods from 1950 to 2005. For cities over a half-million, comparisons of the 1950 data from the two datasets differed by a scalar multiple of 1.182 (indicating that Chandler was consistently somewhat more conservative than the U.N. in estimating larger Metropolitan area populations), with $R^2 = .9985$. Below a half-million size for 1950, up to the size of two million for 2005, we deleted smaller cities from the U.N. data because they were undersampled. Cities were sorted by size and the sorted stacks of city sizes compiled in a single file prior to log-binning by size.

We calculated complementary cumulative populations in cities of 50K up to 25.6 million in multiples of $\sqrt[3]{2}$. By taking cumulative populations we increase the accuracy of curve-fitting and make our graphs easier to interpret visually because the city-size curves tend to rise with time. The binning procedure resulted in distributions of number of people in cities above the minimum for the each of the 28 size bins. In addition, we counted the number of cities in each log-binned interval. If the largest bin containing a city was not full of cities with sizes up to the upper limit of the bin then the bin was dropped from analysis to avoid the effect of unequal bin sampling. Similarly, we dropped cells at the lower end of the binning scale for a given time period if they were not filled to their lower size limit.

Issues of data quality include the changing reliance on archaeological, historical, and census data. The earliest census data come from Han China. Censuses were taken for many world cities by 1700, with data quality increasing after 1800. Even in the earliest of our time periods, however, there are estimates considered to be reasonably reliable from archaeological as well as historical sources.

3 Urban Scaling Through q: Methods and Results

As described in the introduction we follow the modelling of Malacarne, Mendes and Lenzi, and asymptotically compare the estimated parameters q with Pareto CDF. We used the q-exponential function to fit the data of binned city-size distributions from the coding procedures:

$$y(s) = y(0)e_q^{-s/\kappa} \tag{1}$$

$y(s)$ denotes cumulative number of people in cities of size s or larger. Sizes of the cities are expressed in 10^5. The q-exponential function is defined as follows (Gell-Mann, Tsallis 2004):

$$e_q^x = \Big[1 + (1-q)x\Big]^{1/(1-q)} \qquad (e_1^x = e^x = e_q^x,\ q \to 1) \tag{2}$$

[2]The Chandler data were provided by Céline Rozenblat in a text file with columns for city names, empires, city population, latitude and longitude. We intend to redo our analysis with Chandler's 1987 publication of a "vastly improved" and generally more conservative dataset as an independent replication of our results. Another replication will be done against Modelski's (2003) city size estimates. The shape of the distributions might not be changed in major ways but these independent fittings will provide greater opportunities for validation of findings and detection of potential sources of data bias or unreliability.

if $1 + (1 - q)x > 0$ (hence for $q > 1, x = -s/\kappa$ is negative), and zero otherwise. This reduces to the usual exponential function when $q = 1$, but in the limit $x \to -\infty$ it asymptotically approaches a power law when $q > 1$ with the tail approaching a limiting slope $\alpha \equiv 1/(q - 1)$, so that e_q^x approaches $[1 + (1 - q)x]^{-\alpha}$.

We used two different approaches to fit the data.

- The Gauss-Newton iterative algorithm for nonlinear least-squares (NLS) estimates model parameters q and κ and determines the extrapolated value $y(0)$ – the characteristic number of people. Due to noise and lack of the data in some of the earlier time periods the Gauss-Newton algorithm did not always converge or the convergence was not appropriate because the estimate of q was smaller than 1.

- For the second approach we used the $y(0)$ values from the NLS approach. For the data in the earlier time periods with no convergence of Gauss-Newton algorithm we manually added the extrapolated values of $y(0)$. Then we calculated the $\ln_q$:

$$\ln_q\left(\frac{y(s)}{y(0)}\right) = \frac{\left(\frac{y(s)}{y(0)}\right)^{(1-q)} - 1}{1 - q} \tag{3}$$

  for various values of q. Linear regression for points $[s, \ln_q(s)]$ that gives the highest R^2 provides the best estimate of q and the slope of the regression line provides the $(-1/\kappa)$ estimate.

  The q and κ estimates from this approach overemphasize the number of people of the largest cities and therefore give poorer fits than NLS approach. In order to improve the fit we used weighted linear regression (WLR) using number of cities in each bin for the weights. The highest R^2 for WLR varied from 0.92 to 0.99.

Figure 1 shows the city-size distributions resulting from our coding procedures with the fitted q-exponential curves. The x-axis represents the binned size and the y-axis the cumulative population from each bin size up to the largest size. The axes are logarithmic with the y-axis labelled in 10^3 units of city population. In the first two graphs, (a) and (b), the lines were fitted by the NLS approach. For the data where appropriate convergence of Gauss-Newton algorithm did not occur no curves are plotted. For the curves in the graphs (c) and (d) we used estimates q and κ that we obtained from WLR. There are two distributions for the year 1950 due to two different datasets. The data “1950 a” are taken from the U.N. dataset for 1950-2005 and the data for “1950 b” from the Chandler dataset for -430–1950. Because the q value for the dataset of the year 622 deviated highly from all other estimates, and had by far the lowest R^2 of .92, we excluded the data for this period from further analysis.

The dotted black diagonal lines in graphs (a) and (b) represent the line $x = y$ that cannot be crossed by data points according to data binning. The dotted lines were plotted in the way to connect data points with similar cumulative number of cities in each data period. Cumulative number of cities that was used for plotting the dotted lines follow the multiples of two (hence $2, 4, 8 \ldots$). It is the city numbers that are usually plotted in size bins for estimating city-size distributions, but we use them to plot the dotted lines to inspect for the possibility of irregular historical changes between successive periods. If the dotted city-number lines were in parallel for bins of the same size from period to period, it would show that the q-exponential curves (with power-law as a special case) were associated with continuous growth-rates following some city-size proportionality function. This is not the case in figure 1 (c) for such transitions as those for 1800–1825–1850–1900 during the industrial revolution, for example. Transitions in city numbers as well as city populations (i.e., parallelism of dotted lines) are observed 1900 and 1960, but become a bit more irregular after 1960 when world population growth begins to level off. Similarly, there are in figure 1 (d) city-number discontinuities such as those for the fifty-year transitions following incorporation of the new world into a global economy, severe new-world depopulation, and new bullion influx into Europe. Whether these historical variations are autocorrelated or indicative of slow fluctuations or sharper breaks requires testing.

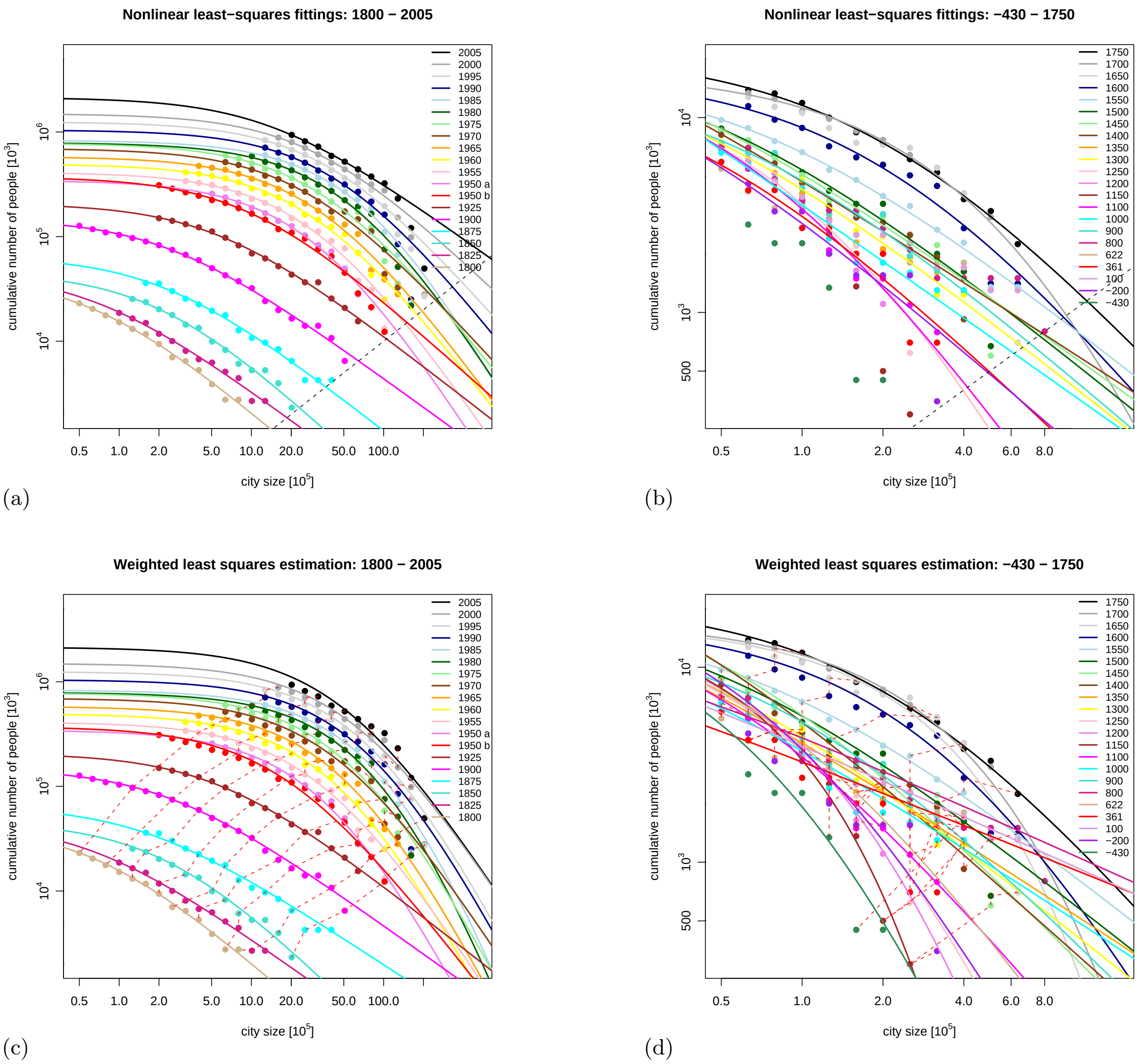


Figure 1: The points on the the log-log scales represent the data (different colors for different years) and the lines represent the fits. Fittings were done using Gauss-Newton algorithm for NLS estimates (graphs (a) and (b)). For the data where the algorithm did not converge, the line is not plotted. The lines in graphs (c) and (d) are made using weighted linear regression estimates of q and κ. Data for the year 1950 are plotted twice (1950 a – from dataset 1950–2005 and 1950 b – from dataset -430–1950). The black dotted line represents the $x = y$ line, which is never exceeded by data points (due to the binnings). Dotted lines on graphs (c) and (d) roughly connect same cumulative number of cities in data periods. It can be seen that the dotted lines cross more often in the earlier time periods which might indicate that the data are noisier or that there are more historical discontinuities.

4 Runs Tests

We studied the 28 q parameters that were estimated by NLS fitting as a time series. We used a runs test (Siegel, 1956) to check for a possible non-randomness (specific time series pattern) in the observations. The test is nonparametric (it assumes no particular distribution of the dataset). It checks if the number of runs (the number of consecutive sequences of same values) is in the correct number for a series that is random. We dichotomized the values according to the mean value of the data and the test rejected the randomness assumption with a p-value of .001. A good example of how q varies by historical periods is the comparison of the period of 1800–1925, where $q \sim 1.84 \pm .09$ (s.d.), with that of 1950–1985 $q \sim 1.55 \pm .09$ (s.d.). The conclusion in this comparison is that the upper-bin urban hierarchy was more egalitarian post-WW II than in the period of the industrial revolution. The trend line for q through all time periods is flat. The κ values, reflecting urban scale and an overall pattern of growth, show a temporal accelerated growth trend toward a singularity in the current century. A runs test for the detrended values of κ shows 10 ± 3 runs and a p-value of .0003 strongly supports the non-randomness in the data.

As shown in figure 2, q and detrended κ show: (1) flatline values between -250 BCE and 1000 CE, (2) negative correlation between 1000 and 2000 CE ($R^2 = .60$), and, for the latter period, (3) runs of high and low values show discrete breakpoints between historical periods. Because we use global data, we believe that (1) indicates that there is no global synchrony in q values prior to 1000 CE. As for (2) and (3), three runs of low q values in figure 2 indicate more egalitarian periods, which correlate with detrended κ values well above the detrended κ average. This indicates that these are periods of high urban population growth. Conversely, three periods of high-q values and detrended κ values below their average indicate hierarchical periods with many urban hubs and lower growth. High or low growth in each case is inferred from κ. Table 1 shows values of $q, \beta = 1 + 1/(q-1)$ (the Pareto equality coefficient for the asymptotic slope where $\alpha = \beta - 1$), κ, detrended κ, and a classification in into low and high q values that we call successive Q-periods.

In order to help visualize the difference between low-q (~ 1.5, egalitarian) and high-q (~ 2, hierarchical city-size hierarchies, from 1000–2000 CE, figure 3 separates the contiguous time periods from the runs test in figure 2 for each of the two types of q values, high (three time periods) and low (three alternate periods). Shapes of these curves are similar within high and low q periods. It is evident visually in figure 3 that the (q_1, q_2)-exponential model studied by Borges (2006) for wealth and financial distributions, if applied to city

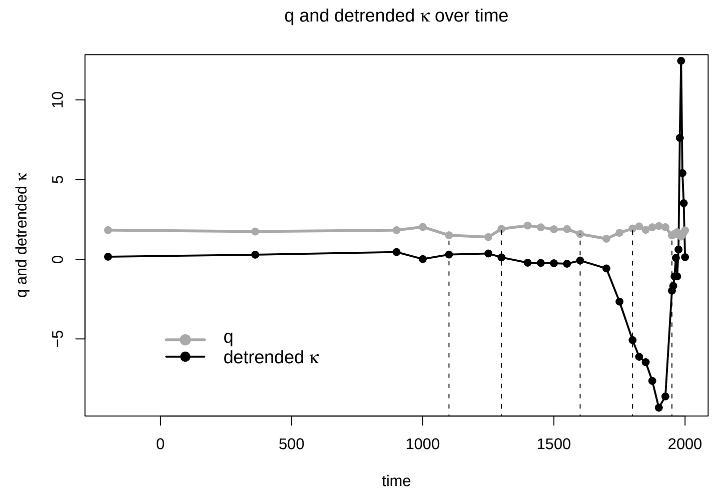
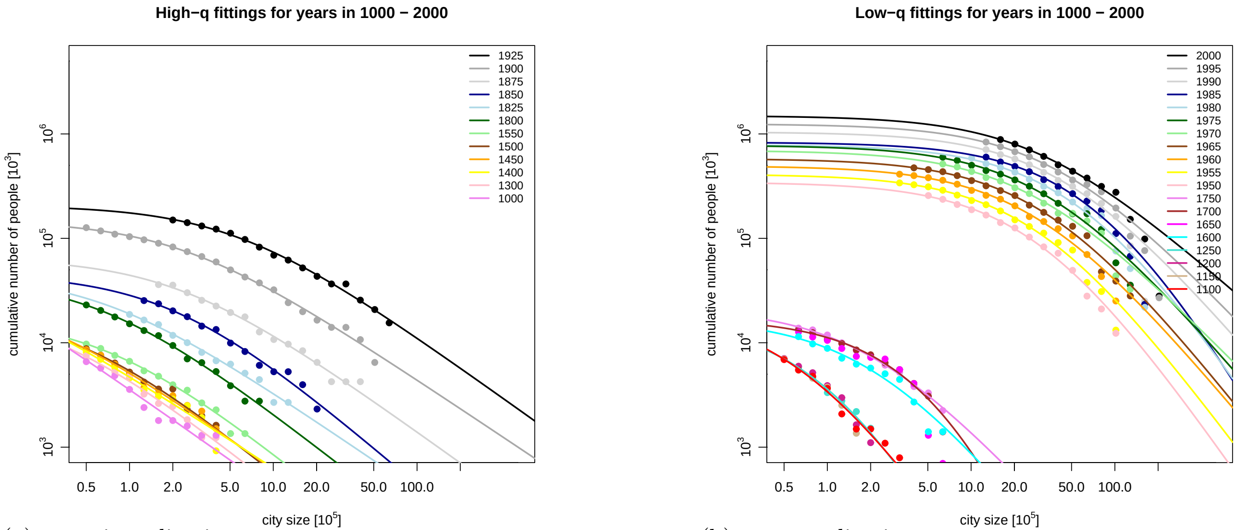


Figure 2: Correlation between q heterarchy and detrended κ resulting in periodization (vertical lines). q and κ were taken from NLS estimation. κ was detrended according to $1.13 \cdot 10^{-5} \cdot 1.007^x$.

Table 1: High-q and low-q values (NLS estimates) for a corresponding time period (from the year 1000 on) and the corresponding parameter κ. High-q values for our CDFs are marked in bold. Values of q_{PDF} and the asymptotic power-law slope $\beta = 1 + \frac{1}{q_{PDF}-1}$ are computed from q_{CDF}. β is analogous to a Pareto equality coefficient (high slope, thin tail). Pareto coefficients lower than one (deriving from estimates where $q \geq 2$) may reflect estimation errors and can be interpretated as low slope, high heterogeneity and therefore inequality.

time period	q_{CDF}	$q_{PDF} = \frac{1}{2-q_{CDF}}$	$\beta = 1 + \frac{1}{q_{PDF}-1}$	κ	κ detrended	Q-periods
2000	1.81	5.26	1.23	24.50	0.13	6
1995	1.67	3.03	1.49	27.01	3.52	6
1990	1.61	2.56	1.64	28.06	5.41	6
1985	1.42	1.72	2.38	34.30	12.45	6
1980	1.48	1.92	2.08	28.68	7.62	6
1975	1.61	2.56	1.64	20.92	0.61	6
1970	1.70	3.33	1.43	18.51	-1.08	6
1965	1.57	2.33	1.75	18.96	0.08	6
1960	1.60	2.50	1.67	17.13	-1.08	6
1955	1.53	2.13	1.89	15.88	-1.67	6
1950	1.50	2.00	2.00	14.95	-1.98	6
1925	**2.01**	-.	0.99	5.49	-8.61	5
1900	**2.09**	-.	0.92	2.43	-9.32	5
1875	**2.01**	-.	0.99	2.16	-7.64	5
1850	**1.84**	6.25	1.19	1.70	-6.46	5
1825	**2.06**	9.	0.94	0.68	-6.12	5
1800	**1.92**	9.	1.09	0.59	-5.08	5
1750	1.66	2.94	1.52	1.28	-2.66	4
1700	1.28	1.39	3.57	2.16	-0.58	4
1600	1.58	2.38	1.72	1.24	-0.08	4
1550	**1.89**	9.	1.12	0.63	-0.28	3
1500	**1.88**	8.	1.14	0.38	-0.25	3
1450	**2.00**	-.	-	0.21	-0.23	3
1400	**2.11**	-.	0.90	0.09	-0.22	3
1300	**1.90**	9.	1.11	0.26	0.12	3
1250	1.39	1.64	2.56	0.46	0.36	2
1100	1.50	2.00	2.00	0.33	0.29	2
1000	**2.02**	-.	0.98	0.03	0.01	1

size data, would show that when we compare the fitted q-distribution curves to the actual data points the larger tails for each period that there is a uppermost slope in the data series that varies from the asymptote in about half of the curves. The tendency is for flatter second-slopes in the low-q distributions and steeper slopss in the high-q distributions but with many of these secondary tails in a direction opposite to the general tendancy. A (q_1, q_2) or two-q model with an added "knee" at q_2, as in the use of q-exponential scaling by Borges (2005) might well be justified (but we do not attempt this here). The flatter-tails above these additional "knees" might be typical of primate city outliers, with thinner extreme tails in the upper power-law tendency. These are common in the high-q periods. Steeper slopes above such "knees" might indicate the exteme urban hubs that are especially common in low-q periods and that are also centers of innovation and population growth.



(a) more inegalitarian

(b) more egalitarian

Figure 3: Fitted q-distributions for 1000–2005 CE are repartitioned into high-q (Eastern Roman Empire, Renaissance depression, Industrial Revolution) and low-q (medieval Renaissance, Early modern, Post WW II).

5 Validition-check against Economic K-wave and Political Leadership (L-Cycle) start and end dates

Because we had no inkling *a priori* that our city data would scale into periods we also had no prior expectation about oscillitory periods or that the periods that emerged from our analysis would correlate with Schumpeterian, Modelskian, or any other prior findings about long-term historical wave phenomena. We completed our scaling of q-exponentials for our 28 historical periods before examining and testing the start and end dates of our q-periods against the shorter economic and political cycles identified by Modelski and Thompson (1996). For our validation test we thus have two independently derived data sets: our q periods, on the one hand, and the start/end dates of leading economic sector innovation periods (K-waves) and globally leading nations (L-cycles), which Modelski and Thompson find to be intricately entwined.

K-waves are not sine waves but S-shaped logistic growth curves of innovation outputs for a leading economic sector. Modelski and Thompson, citing 1000 CE as the onset of a Eurasian globalization that has lasted until today (which accords with our finding from figure 2), identify 19 such spurts of economic innovation in Eurasia (K1-K19). Since 1430 it has been possible to identify two phases within each K-wave, ones which Berry (1991:161) identifies with Kuznet's construction/immigration spurts to leading sectors, but which Modelski and Thompson identify with interwoven phases of economic and political cycles. On longer time-scales, two K-waves typically occupy a single global world leadership (L- or long) cycle, the first K-wave marking the rise of one power (through technological or economic innovation) and the second K-wave marking a challenge from another economic sector and polity and concomitant decline of the first power. The world political leadership is replaced in each L-cycle, of which there are ten (LC1–LC10). As shown in table 2 (first panel), sets of four K-waves form repeated economic "long learning cycles" (LLCs) that lead to economic breakthroughs.[3] While single long learning cycles (LLCs) are formed two K-waves (panel 1 of table 2), the L-cycles as we display them in panel 3 of that table shows doubling again for some periods in a Q-period with two phases demarcated within the K-waves.

Table 2 summarizes changes in Q-periods (our high- versus low-q periods) as against the complex interweaving between the sequence of leading political powers as it tracks economic leadership. We find that Q-periods last much longer than L-cycles, on average twice as long, and very much longer than K-wves, on average four times as long. Thus may represent a previously undiscovered in historical long wave phenomena, especially since they gain validation by the fact that their oscillations interweave quite closely with known phenomena. The correlations between Q-periods, L-cycles, and K-waves, however, are neither perfect nor mechanical. We will attempt to understand here something of how their timings and embedded processes interact through time.

Table 3 provides the specific dates for each K-wave, L-cycle, and Q-periods. How these cross-cut one another evolves over time with long-term learning. The LLCs in sets of four tend to correspond to our Q-periods, only substituting 3–4–1–2 for their order of phases 1–4, as in the second panel of table 2: (1) basebuilding for a competitive breakthrough, (2) network building, (3) the innovative economic breakthrough itself, and (4) payoff of the breakthrough via distributive networks. We call reordered 3–4–1–2 sequences economic long waves, *eLW* for short. But while the first two of these four-phase eLWs alternate between low- and high-q urban scaling regimes, the correspondence is slightly off in the last three (phasing as 3–4–1–2–3, and then as 4–1–2–3 and 4–1–2–3).

The more remarkable property for our study is the almost perfect correspondence, shown in the third panel of table 2, between our Q-periods and a reordering of what Modelski and Thompson provide for the phases of LCs or the long cycles of leadership, which are slightly different than the phasing of economic LLCs. Our panel three shows their a–b–c–d–a–b ordering, which goes from agenda setting (a) to coalition building (b), to macro-decision (c) to a power confrontation through global war, then to a successful execution (d) of the war to become the leading world power, followed by delegitimation as a contending power arises (back to a), and finally deconcentration (b) as the new power builds its coalition. If we start with the global power emerging as the winner of a global war (c), and follow the sequence through its four phases, c–d–a–b, we find an almost perfect correspondence with our roughly constant Q-periods. This correspondence has two stipulations. The first is that all four of the K-wave phases (in one case, five) in an eLW or LLC are executed, at least up to 1914. The exception to this stipulation occurs when the leading power (Britain) exhausts its strength in winning the global war, and its ally, the United States, fills in the gap. Then, after WW II, the U.S. takes Britain's place as a leading power. In all cases, however, the LLC stage 4 – payoff – comes early in each Q-periods, which entails that each initial leader of a new Q-period comes equipped with economic success. A corollary is that each Q-period begins with a stable rather than an inflationary world economy. Further, in all cases, the LLC stages 1 and 2 are toward the end of a Q-period, which entails the rise of a contending power with competitive economic clout. A corollary seems to be that the form of economic organization utilized by a contender is often countercyclic or contrarian to the form established initially.

An alternate stipulation is to state, as a rule, that a Q-period goes through two or even three leading polities and is broken only with a global war in which the outcome shifts the power balance, as for example, with the wars of the late 13th century that spelled economic decline, the Dutch-Spanish wars that put the

[3] Modelski reports that the hypothesis of one period of evolution in the global economy made up of two global leadership cycles and four K-waves per period was unsupported by systematic data but supported by general knowledge of economic conditions. This theoretical prediction thus seems validated by evidence for q-distribution.

Netherlands in a financial position to aid economic recovery (followed by the industrial revolution), the defeat of Napoleon that allowed a global spread of maritime colonial powers, and the defeat of Germany's early 20th century bids to industrial and colonial hegemony. A corollary seems to be that the initial globalization and these global wars that alter Q-periods seem to mark alternating global upturns (Sung to North Italian Medieval Renaissance, the Dutch and British financial and industrial breakthroughs, and Post-WW II prosperity) or downturns (the long "Renaissance" depression, and the long depression of the Enlightenment).

For us, these results were unexpected: we had no preconceived idea that our results would correspond in this way to sets of Schumpeterian K-waves or Modelski L-cycles. If we take a closer view of the medieval periods in Sung China, splitting Northern from Southern Sung, and take city-states as the political unit in Northern Italy, distinguishing the early Venetian versus later Genoese hegemony, we see that political growth cycles take place within contrastive high- versus low-q city size periods throughout the millenium (shown in table 3). The evolution goes from small scale (e.g., city-states) to medium (e.g., nation states) to large (e.g., continental-scale polities). Q-periods are underlying constants for each of the temporal eLW cycles that involve global political and economic contexts.

More detailed validation of how Q-periods form a substrate for the embedding of political and economic processes in urban size distributions and dynamics, and how change of Q-periods are affected by the latter, can be carried out by examining the mobility from period to period of specific cities in relation to the leading economic sectors and polities. As expected, for example, Chinese cities and those along the trade routes to the Mediterranean are foremost in the early Q-period distributions. European cities are seen to rise from the middle part of the Medieval renaissance (a low-q period). Detailing these patterns must await a separate publication.

[Insert table of the major urban hubs of the four Q-periods]

Figure 4 helps to visualize the relation between K-waves of leading economic innovation sectors, (labelled K10-K19 and approximated in time-scale by the Kuznets cycles of circa 55 years), L-cycles of political leadership (LC6-LC11, dotted lines rising then falling in each "national" period of leadership), and Q-periods (Labelled Q4-Q6).

6 Network-Economic Generative Theory for Urban q Oscillations

It was only after finding evidence of urban oscillations that the comparisons against Modelski and Thompson's data were made. We had already done our network analysis of intercity network dynamics in the first half of the millenium and articulated a theoretical model to help explain our results. We can note now that there is nothing in the Modelski and Thompson data on K-waves and L-cycles that would predict Q-period oscillations. K-waves and L-cycles go on unabated and unchanged in general pattern through both types of Q-periods, high and low. The theory that we developed independently, before our validation tests, thus addresses an important question that remains unexplained: why the difference in Q-periods over much longer time scales than L-cycles and K-waves?

It is tempting to model the oscillations in q in figure 2 by fitting some rather simple mathematical functions, such as Lotka-Volterra oscillations. Alternately, it is tempting to consider these oscillations as two long-lasting but instable equilibria that can kick over to the other equilibrium given sufficient internal perturbations.

A good theoretical model, however, should consist of solid and testable derivations and be tested by measurable variables. The variables of a theoretical model for relative city growth must include or reflect measureable changes in industries, commerce, and finance, how they are interconnected by networks of cities and hinterlands, and how their distribution and flows affect migration up and down the city-size hierarchy. In the melange of economic variables, transport costs in intercity networks are critical to how economic flows in networks will be configured, and here it is necessary to include the effects of political conflicts, both internal and external, as they affect transport network infrastructure (e.g., destruction or building of roads and bridges) and safety. The complexity of these interactions does not end here, because organizational variables (such as the building of bridges and roads in the medieval period), organizational transformations

Table 2: Correlates of Q with Phases in Long economic and political leadership cycles (Modelski and Thompson, 1996 (MT)). High-q periods are inegalitarian, low-q periods are more egalitarian periods.

Phases of economic cycles Modelski and Thompson (1996:132)	Base bldg.	Network	Innovation Breakthru	Payoff	city size distribution
(Long Learning Cycle:)					
Phases of economic breakthroughs to...					
LLC1 National markets (Sung)	K1	K2	K3	K4	high to low q
LLC2 Commerical-Nautical revolution	K5	K6	K7	K8	low to high q
LLC3 Oceanic trading	K9	K10	K11	K12	high to low q
LLC4 Market economy	K13	K14	K15	K16	low to high q
LLC5 Information economy	K17	K18	K19	K20?	high to low q
eLW: economic Long Wave (q-reordered)	Breakthru	Payoff	Base (new)	Network	(new)
1 Silk Road to Sung Breakthrough	-	-	K1	K2	high-q
2 K2 Sung Breakthrough to Black Sea	K3	K4	K5	K6	low-q
3 K6 Black Sea to Indian spices	K7	K8	K9	K10	∼high-q
4 K10 Indian spices to Amerasian trade	K11	K12	K13	K14	∼low-q
5 K14 Amerasian trade to Automobile Age	K15	K16	K17	K18	∼high-q
6 K18 Automobile Age to Modern economy	K19	K20?	K21?	K22?	low-q
Phases of polity Leader L-cycles New Leader Ascent (a,b,c,d): Thompson (2000:87)	c Macro decision	d (world power) Execution	a Agenda setting	b Coalit. Building	MT (1996:8) city size distribution
LC1 (none)/No. Sung			K1	K2	high-q
LC2/3 So. Sung/Genoa	K3	K4	K5	K6	low-q
LC4 Venice/Portugal	K7*	K8	K9	9b	high-q
LC5 Portugal/Netherlands	K10	10d	K11	11d	
LC6/7 Netherlands/Brit.I	K12*	12d	K13	13b	low-q
LC8 Britain I/II	K14	14d	K15	15b	
LC8 Britain II/U.S.	K16*	16d	K17	17b	high-q
LC9/10 United States/?	K18+*	18d	K19	19b	low-q
Former Leader Decline (d,a,b,c)	Global war	New power	Delegit.	Deconc.	
*Globalization and Global wars affecting Q-period transitions	Resulting Leadership	K-Wave	War	Effect on Land	MT (1996:54)
∼1000 Onset: Oceanic Globalization	No.Sung	K3		Safe	
Y 12XX-1279 Mongol conquest of So. Sung	Mongols		Regional:Land	Unsafe	
” XXXX-1298 Genoese Naval Victory	/Genoa	K7*	Regional:Sea	”	
N 1494-1516 Wars of Italy/Indian Ocean	Portugal	K10	Global:Both	Safer	
Y 1580-1609 Dutch-Spanish Wars	Netherlands	K12*	Global:Both	Safer	
N 1688-1714 Wars of the Grand Alliance	Britain I	K14	Global:Land	Safe	
Y 1792-1815 Wars of Fr. Rev./Napoleon	Britain II	K16*	Global:Land	Unsafe	
Y 1914-1945 WW I and II	U.S.A.	K18*	Global:Both	Safe	

Table 3: Dating correspondences between q-exponential periods, K-wave leading economic sectors and leadership (L-)cycle hegemons from Modelski and Thompson (1996:137). Leading sector waves are shortest; polity hegemon waves longer; and urban heterarcy/inegalitarian oscillations the longest. Asterisks show periods of world wars and naval buildup. For Q-periods high-q means hierarchy, many hubs and low-q stands for equality and few hubs. Comments in LC column 6 indicate suggestive data for alternation of financial and commercial capital in different sectoral configurations and time periods.

(startup) Dates	L-cycle: Leading Nation (Modelski)	K-wave: Leading Sector (Schumpeter/Kondratieff)	K-wave sector	2-phase L-cycles for hegemon (also forms of capital)	Q-periods (q-hubs: few or many)
		(silk roads)			Q1 (pre-930)
930	No. Sung	printing and paper	K1	LC1 Commer.	” high-q
990	”	national market formation	K2	”	” high-q
1000	Onset:	(globalization)			”
1060	So. Sung	fiscal and admin. framework	K3	LC2 Fin.Admin.	Q2 low-q
1120	”	maritime trade expansion	K4	”	” low-q
drop?	→ Venice	corporate “eastern” shipping	-	- Fin.Corp.	” low-q
1190	Genoa	Champaigne fairs	K5	LC3 Commer.	” low-q
1250	”	Black Sea, Atlantic trade	K6**	”	” low-q
1300	Venice ?	galley fleets	K7	LC4 Fin.Corp.	Q3 high-q
1350/5	”	pepper	K8	”	” high-q
1430	Portugal	Guinea gold	K9	LC5 Commer./Org.	” high-q
1460	”	”	9b	”	” high-q
1494	”	Indian spices	K10*	”	” high-q
1516	”	”	10d*	”	” high-q
1540	Netherlands	Baltic, Atlantic trade	K11	LC6 Rel.Corp.	” high-q
1560	”	”	11b	”	” high-q
1580	”	Asian trade	K12*	” Fin.Corp.	Q4 low-q
1609	”	”	12d*	”	” low-q
1640	Brit.I/Colonies	Amerasian plantations (tobacco,	K13	LC7 Commer.	” low-q
1660	”	” sugar, rum, slaves)	13b	”	” low-q
1688	”	Amerasian coffee, sugar, tea	K14*	”	” low-q
1714	”	”	14d*	”	” low-q
1740	Brit.II/Industry	cotton/textiles, iron	K15	LC8 (Dist.Fin.)	” low-q
1763	”	”	15b	”	” low-q
1792	”	steam, railroad	K16*	”	Q5 high-q
1815	”	”	16d*	”	” high-q
1850	United States	steel, chemicals, electrics	K17	LC9 Commer.	” high-q
1873	”	”	17b	”	” high-q
1914	”	autos, air, electronics	K18*	”	” high-q
1945	”	”	18d*	”	”? high-q
1950					Q6 low-q
1973	United States?	information industries	K19	LC10 Fin.Corp.	” low-q
2000	”	”	19b		” low-q few
∼ 2026?	?	?	K20?	?	”? high-q?
∼ 2080?	?	?	K21?	LC11 Commer.?	”? high-q?
City size Q	∼ 200 ± 20 yr.	oscillations			
	Pol. hegemony H	∼ 110 ±15 yr. oscillations			
		Economic sector innovation K	∼ 55 ±10	yr. oscil.	
Cross-Tab:	K-wave A (sets of 4)	K-waves B (sets of 4)	L-cycle sets	L-cycle sets	
high-q	8	2	14	0	
low-q	3	6	1	13	

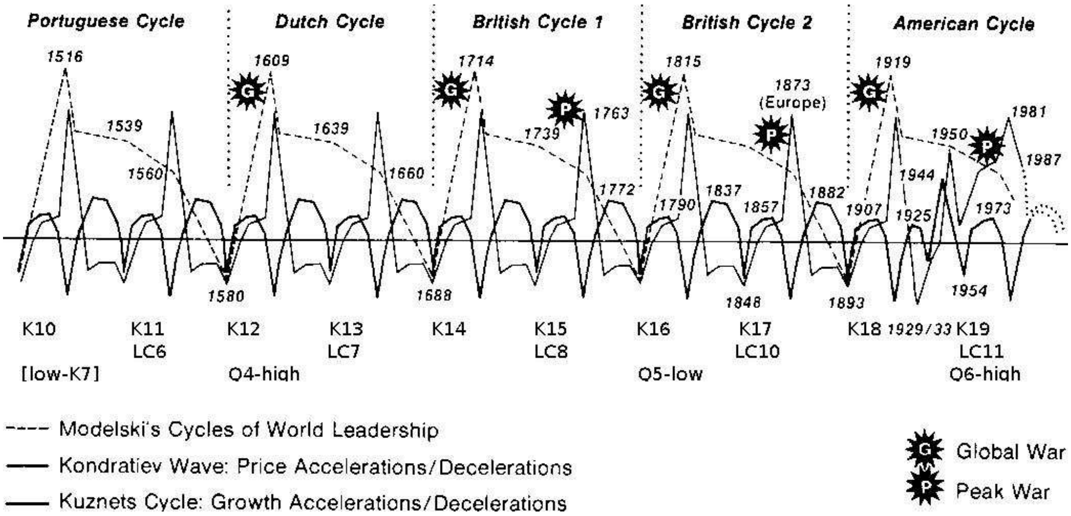


Figure 4: A schematic modified from Berry (1991:161) to show the nesting of construction and labor migration (Kuznets) cycles, inflationary (Kondratieff) cycles, world political leadership (Modelski) cycles, and q-cycles. Schumpeterian K-waves are not shown but operate at the time-scale of Kuznets cycles.

(e.g., the emergent organization of trade by Hanse religious brotherhoods in northern medieval Europe, or the scaling-up of particular merchant organizations and agents with increases in monetization and the velocity of trade) enter the equation.

The idea here will be to see whether an intercity trade network is easily navigable by traders with no global knowledge of its network structure. We take as given the q-exponential city size hierarchy of the network, and the distribution of the typically low number g of roads or shortest port-to-port sailing routes in current use by each city. Because we find in intercity network data for the medieval period (White and Spufford 2005) that the effective g for different cities is a long-tailed power-law function of s, we denote this expectation is g_s. Whether the network is considered to be easily navigable from a given city to a random target is measured by the probability of success of navigating from an average city to the largest hub in its neighborhood, and then iteratively to the next largest hub of that hub, until a hub is reached that has the target in its neighborhood. This navigability criteria was defined by Adamic, Lucose and Huberman (2002) in the study of scale-free networks. This type of hub navigability depends on network characteristics which make the number of links from source to location of a target very low in comparison to random search. Applied to trade navigability in a network of cities, the idea is that a merchant or agent could carry goods from hub to hub looking for an exchange partner that has the desired target goods to offer in exchange. Climbing the hub hierarchy, if there is a suitable gradient, the trader will eventually find a hub that either satisfies the goal or that has a smaller city in its network neighborhood that satisfies the goal.[4]

[4]Adamic, Lucose and Huberman (2002) investigate local searchability in scale-free networks, in which hubs are those with the most links and the probability of a node having k neighbors is proportionally decreasing only in relation to a power, α of k, $\sim k^{-\alpha}$. They find that when $\alpha > 2.3$, the thin upper tail of hubs is so sparsely populated with nodes that most nodes lack local connection to a hub, so that an attempt to search the network by moving to the most connected node through the hubs of successive neighborhoods will not reach up the hub hierarchy and will fail to locate a target in a number of moves that is significantly more efficient than a random search. When the connectivity hierarchy of a network has a sufficiently thick tail (hierarchical), in the range $2 \leq \alpha \leq 2.3$, a network is navigable through successive hubs; but if $\alpha < 2$ the network becomes unsearchable because of too little hierarchy. The importance of the navigability model for social and economic networks is

To enlarge the network aspects of a provisional theory, then, we consider how the global q characteristic of city-size hierarchies affects a global intercity network distribution $r(s)$ as calculated in equation 4. It gives for each city-size (which is denoted by s) the average size of its largest neighboring city.

$$r(s) = \frac{1}{|C(s)|} \cdot \sum_{c \in C(s)} \max_{n \in N(c)} (s_n) \tag{4}$$

$N(x)$ represents the set of g_s closest neighboring cities of x and $C(s)$ represents the set of cities of size (binned to size) s. The average of $r(s)$, an average hub adjacency ratio, can be computed in the following manner:

$$r = \frac{1}{\sum_s |C(s)|} \cdot \Big[\sum_s r(s) \cdot |C(s)| \Big] \tag{5}$$

or

$$r = \frac{1}{\text{no.of bins}} \cdot \Big[\sum_s r(s) \Big] \tag{6}$$

[NK: The last two equations give different results and I think it's good to mention just one - the one which suits our purposes better.]

We assume that the function $r(s)$ is first increasing and then for largest s decreasing. In a low-q or thin-tailed urban distribution, $r(s)$ should be small because average pairs of cities chosen randomly will be more similar when urban hubs are rare. In a high-q thick-tailed distribution, $r(s)$ should be large for smaller s because of the greater heterogeneity of sizes of random pairs of cities. The average r should also be larger than for a low-q distribution. This average should reflect the entire city-network economic structure in terms of navigability. Smaller cities in an inegalitarian size distribution may have trading advantages over those in a low-q or egalitarian city distribution due to differences in the slopes of $r(s)$, depending on whether they have a sufficient gradient of urban hubs in their neighborhoods to allow successive navigation from hub to hub as an effective trading strategy.

The first theoretical derivation of this model, then, is that high-q cities with sufficient hierarchy to form a navigability gradient – a condition we will call *heterarchy* – are able to operate through a commercially based form of capital and exchange given hubs in local-neighborhoods whereas low-q periods cannot, and will be more dependent on credit and financial capital. Thus, employing the network variable r along with q, we can articulate a provisional theoretical explanation for the alternation between the two historical regimes exhibited by q.

A second derivation follows simple economic principles. Assuming low-q relative urban equality with a thin-tail consisting of a few hubs able to engage in long-range financial and credit transactions, we might expect fairly uniform pricing equilibria in exchanges, long or short, assuming that there are multiple paths between very pair of nodes in the network to facilitate fair exchange. This can only occur, however, in the absense of conflicts that interfere with trade and block trade routes. A low-q (egalitarian)period of city-size distributions ought to be more economically efficient thus more productive than one that is high-q, and consequently should be marked by high population growth over all sizes.

A high-q period, however, operating through local transactions, will tend to have hubs at multiple scales. Trade in commodities would accrue greater advantages to hubs with greater betweenness centralities along shortest paths. Conflicts might simply cause deviations in the shortest paths as some would be blocked. This is an urban hierarchy that might be relatively stable even with large-scale landed conflicts. Such conflicts, alongside economic inefficiencies, would also lower population growth. Lower population growth would add to stability by relieving population pressure on resources.

As Krugman (1991) notes, there is nothing wrong with circular arguments in economic geography, even if they fail to lead to a stable equilibrium outcome. Each of our two types of historical period might be relatively stable but also marked by a different type of long-term unsustainability. Here, we have to rely on our observations about historical variables over the last millenium. These are derived from Spufford's (2002)

reviewed and explicated by White and Houseman (2002) and White and Johansen (2005).

detailed study of the European economy in the medieval period, and an extensive compilation of historical and network variables over the last millenium, predominantly for Europe, by White and Spufford (2005).

Benefits of credit and finance capital, relatively free trade, the invisible hand, and keeping polities out of interference with the economy would seem to be applicable in a low-q (egalitarian) urban period if there were controls on conflicts among polities. If a low-q (egalitarian) urban period could be stabilized, it would be through the reduction of international conflicts. Such periods have the potential for long-range economic dynamism and development through production, imports and exports, and long-range borrowing of ideas and organizations. The population growth that is enabled is a mixed blessing, however, and left unchecked, a source of instability. One possibility is that the upper-sized cities, whose growth may be accelerating according to a leading factor, $dx/dt = x^q$, are able to innovate at a rate that supports accelerated growth and that attracts (e.g., migrant) populations diminishingly from cities of lesser size. But while faster population growth in low-q hubs supported by innovation and proliferation of economic functions of hub cities, such growth cannot continue forever and is a source of eventual instability.

Judging from our historical European examples over the last millennium, the low-q (egalitarian) urban period is unstable in the long run because of competitive political interventions and land-based conflicts that are disruptive to land-based trade. These become more likely as larger political agglomerations form and as their centers become aware of the trading inequalities and opportunities offered by long-distance trade. As polities develop economic opportunities offered by political interventions at these longer distances, outside the normal localized trading webs, political influences that alter or disrupt the longer-distance terms of trade become more likely.[5]

As shown in the lower panel of table 2, global wars tend to occur at the transitions between Q-periods. In some cases, the outcomes and subsequent conflicts create instabilities for landed trade (Q2/Q3, Q4/Q5) but in other cases the outcomes create stable alliances that insure the subsequent safety of landed trade (Q3/Q4, Q5/Q6). Modelski and Thompson (1996:54) show that each L-cycle involves global war as the third phase (Macro-decision, see the third panel of table 2). Outcomes for stability, in their view, stem from the quality of the leadership established by the world powers that emerge from these conflicts.

When disruptions to land-based trade become sufficiently serious in a low-q period, a shift to port-based trade and maritime trading becomes more likely because these offer lower transport-cost alternatives by avoiding land-route conflicts. Many of the negative characteristics of the high-q hierarchical periods may be due to the international conflicts that inaugurated them and then characterized their regimes. Many suboptimal politico-economic policies, such as printing money, can also be traced back to state financing through loans of international conflicts. If the conflicts among polities could be regulated, these periods might be optimal for sustainable human habitation of the planet, as they maintain slower and potentially sustainable population growth. But under peaceful conditions better and broader credit and financial institutions will evolve that give way to low-q periods. The high-q period is unstable in the long run as the great cities, especially maritime trade centers, actualize their potential to become commercial centers. New forms of economic productivity and the skilled populations that support them develop in the profit centers. These developments can quickly bring about a shift to a low-q (egalitarian) period.

High-q or inegalitarian city periods appear overall to be ones of economic depression. But in Europe the great economic depression of the late medieval period goes under the name of the Renaissance. The great cities within a less differentiated city size hierarchy possessed considerable potential for autonomy that promoted cultural and intellectual developments, and affected through diffusion the larger populations, as in northern Italy or the Low Countries. Similarly for the period of the later European period of the Enlightenment, which was also a high-q hierarchical urban period that corresponded to a long-term economic

[5]It is not apparent to us whether long-run instabilities in low-q periods might be generated by rising inequalities in the interurban terms of trade that are driven by the lack of sufficient heterogeneity in the city hierarchy so that hub-navigation can occur as described for high-q periods. This might occur because this inhibits the ability of an average city to engage successfully in local commercial trade. If trade comes to be predominantly mediated through the larger urban hubs, with longer distances between traders in smaller cities and those in the large hubs, the terms of trade for smaller cities (often mistaken for the rural/urban terms of trade) may become markedly unequal in relation to hubs, and because only the much larger urban hubs can easily afford the costs and can benefit from scale-efficiencies in long-distance trade. Economic inequalities could increase dramatically in the long term as a consequence, leading to civil conflict and interregional disorder.

depression. And while these depressions started out with relative price stability, they gave way to long runs of inflationary economics in many regions.

Both high- and low-q periods ought to be affected by inflation. With inflation, the value of property goes up while that of labor goes down. Populations growing faster than resources produces inflation. Population growth thus may indirectly feed inflation and thus wealth inequality. As the slope (α) of a power-law inequality falls (higher-q) for income or wealth, for example (see Newman 2005), a smaller fraction of units in the population take up a larger portion of the resources (top cities more population, richest citizens more wealth, etcetera). If low-q (egalitarian) urban periods are also high growth, they are also subject to internal conflict as a source of instability, as noted by Turchin (2002, 2005, 2006). These conflicts could also emerge within specific polities. Goldstone's (1991) Demographic Structural theory of the inability of states to govern effectively in the face of population growth becomes highly relevant to the collapse of individual states with concomitant growth of economic inequality. Inflation is one of the major factors affecting the dynamics of wealth inequality. Depending on the correlation between q in size regimes and α in wealth regimes, rising inequality in wealth may also be a factor promoting internal conflict within polities.

7 Interpreting Historical Q-Periodicity

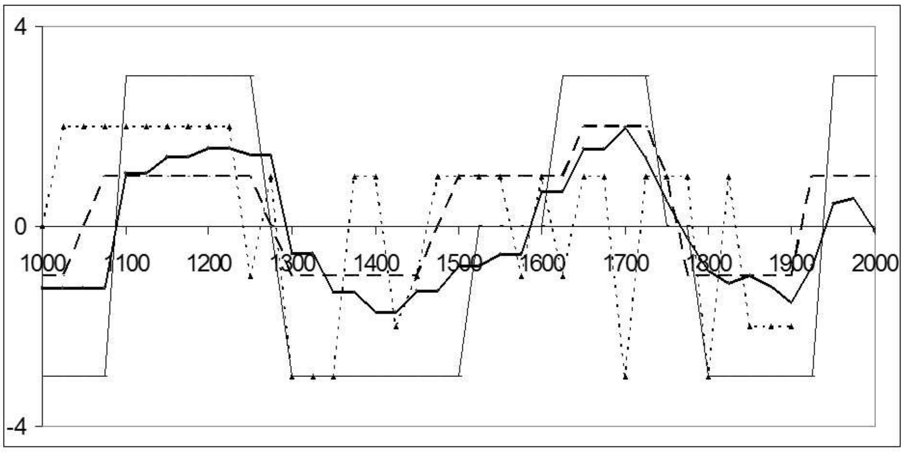


Figure 5: **[NK: Here also I can make a figure that will match the style of the others if you give me the data. DW: GM suggests there are too many lines in the figure, we should split the ups and downs into two figures, and the Far Eastern power distributions may not be the most relevant because of the merging of China and Japan in the global system by 1500.]** The heavy line indexes reversed-variations in q (processes generating low-q equality) for world cities plotted by historical time along with two European historical correlates. The dashed line shows approximate periods of European land trade route safety (absence of international wars); the light line shows the likelihood of alternating European economic periods dominated by financial capital operating through banks and corporate organization (upper values) versus commercial capital operating largely through trading diaspora (lower values). Land route safety tends to lead changes in periods of financial capital accompanied by q equality. Changes in political form in the Far East are indexed by values along the dotted line corresponding to Wilkinson's (2004) coding 2=multipolar, 1=unipolar, 0=tripolar, -1=bipolar, -2=hegemonic, -3=unified empire.

Eurasia, from Europe to China, contributes over 95% of the large world cities for all time periods. As Modelski and others have shown, there is considerable synchrony within Eurasian trade and conflict networks. If K-waves are generated by leading economic innovations and L-cycles by consolidation of innovative political hegemons, how are we to understand Q-periods, two of which together make a Q-wave? We look here, starting with figure 5, at some additional data to modestly test our network-economic generative theory concerning Q-periods. Our generative theory predicts low-q egalitarian periods with financial centers among the thin q-tail of hubs, and that these financial hubs serve to level the general playing field of cities, facilitating flows at longer distances, only when land routes are safe.

East and west Eurasia have different but roughly synchronous patterns after 1000 CE. Two historical correlates of **reversed variations in** q for Europe are shown in figure 5: approximate periods of land-route trade safety (Spufford 2002) and the likelihood (not yet firmly established but see notes in column 6 of table 3) of alternating economic periods dominated by financial capital operating through banks and corporate organization versus commercial capital operating largely through trading diaspora (Spufford 2005, Arrighi 1994). The results fit our generative theory, at least for west Eurasia, where in periods of safe land-based trading (absence of international wars and brigandage), facilitated by financial and credit centers, there are networks of cities mostly similar in size but with some extreme hubs (low-q). These periods tend to alternate with high-q periods that are also those of conflicts that are interruptive of trade so as to raise the transaction cost of land-route trading. With disruptive land-route conflict, trading advantages accrue through shortest (safe) paths. White and Spufford (2005) show that under these disruptive conditions, commercial wealth tends to accrue through short-path inter-urban network centrality. This is a likely factor leading to extremes of urban size and wealth heterogeneities (fat tails) that raise inequality. More locally oriented commercial trading by hubs in the tails of the high-q period, already competitive in the earlier low-q period, come to dominate. An example for Europe of these fluctuations is the transition from a hierarchical high-q period up to 1100 to an egalitarian urban low-q period with a financial center in Venice. The egalitarian period constitutes a "medieval Renaissance" of about 190 years but with Genoa as a countercyclic commercial center challenging and winning naval supremacy from Venice, and then losing to Venice. As Spufford [2005] notes, financial centers tend to outlast their regimes because the networks are translocal rather than dependent on trading links per se, as are industry and commerce. Another period of landed conflict begins in the late 13th century and marks the end of the European medieval Renaissance.

An alternation between financial and commercial forms of capital along with fluctuations in q, as shown for Europe by the lighter of the lines in figure 5, does not hold for China. The Far East has some parallels, however, in forms of states and empire, with integrative corporate extractive periods in the medieval Sung period (with Liao-Jin occurring in tri- and bi-polar parallelism, where polarities refer to the number of contending major states or empires) and early modern (Qing) periods, alternating with the more dynamic but inward-turning Ming commercialism (1368–1644), in which maritime trade is restricted, and finally, the disruptive period of the late Qing dynasty and aftermath of the Opium Wars. Communist China returns to a more corporate and centralized organizational form. Alternations in Far Eastern forms of states and empire are shown in figure 5, as coded by Wilkinson (2004), as transitions in the dotted line starting from tripolar (three large states, 1000 CE) to bipolar (coterminous with the Sung period) to bipolar (1250) and unipolar (1275) followed by a brief period of unified Empire under the Yuan (Mongol) dynasty (1223–1356) in their period of maritime trade expansion. The curve traced by these changes follows somewhat that of Europe in its implications. The corporate form involves competition between states while the European disruptions of land-conflict and more decentralized commerce are paralleled by the kind of centralized exploratory commerce organized by the Sung empire. This is followed by a roughly 50-year period of unipolarity with rivals to the empire, then a single period of rising Ming hegemony (1425), and a long period of oscillation between unipolarity and bipolarity shared with rivals in the period of Ming isolationisms. China, of course, had a quite different set of political issues than Europe with the rivalry between agricultural states and large scale political pastoralism, with the Golden Horde operating the westward silk road trade in the late medieval period, and the Qing conquest and dynasty as a challenged unipolar state (1650–1825) with brief unification (1700, 1800). The centralized Qing period up to 1825 is one that corresponds roughly to Europe's early modern land-trade financial organization, and the degenerate phase of the Qing after the Opium Wars corresponds to the next alternation in Europe to industrialism. The more general interpretation for Eurasia, then, is that the more hierarchical high-q periods are likely to be more conflictive or decentralized, alternating with low-q urban economies that are centrally coordinated, whether through banks and corporations or through expansive states and empires.

8 Integrating Networks and Innovation Theory

Comparing our theorizing about network structure in Q-periods with Modelski and Thompson's theorizing on innovative leading sectors and leading nations, elements may still be missing that are needed for an integration that draws on historical dynamics. One such element is the extent to which the leaders of one period draw on the innovations of the previous period. Consider the innovations coming from China in the silk-roads period from 100 CE to 930CE, and then the Sung development of printing and paper, credit and accounting systems, and maritime and market innovations. All of this diffused into the Mediterranean to facilitate the early medieval renaissance of the Italian city states. As regimes fall, their innovations and the potentials of their technological infrastructures remain.

A second example, omitted by Modelski and Thompson, is the "invention" in the early stages of Italian city state development (borrowing from Arab and Far Eastern precursors in the organization of long-distance trade, from Cairo and Basra to Guangdong, China, for example) of a corporate financial structure for their trading voyages, one in which craftsmen and entrepreneurs from Venice invested in shares of a profit-making corporation associated with the very ship in which they would embark on a trading voyage. This innovation, from an inegalitarian Q-period of network hubs, shows up in Modelski and Thompson's (1996:19) account in the period after their defeat by Genoa in the first instance (1298), a period in which they are a contender against Genoa (which remains dominant) and well before their return to defeat Genoa in the second great naval battle of 1380. Between 1300 and 1320, after their defeat at sea, the Venetians increased the carrying capacity of state-build merchant galleys seven-fold, organized them into convoys equipped also with oars, and required all convoys transit through Venice.

In comparing examples such as these to Modelski and Thompson's data in table 3, we discovered that our network theorizing, in assuming that the global network structures of Q-periods would reflect leading practices in the same period, was still mired in functionalism. What we learned instead was that the leading practices of one Q-period anticipate the network structure of a succeeding period. Thus, the innovation of centralized galley fleets by the Venetians occurs within the egalitarian Q-period still dominated by Genoa, which as period dominated by economic competition and conflict on land, is one in which the most centralized of the maritime traders – Venice, then Portugal – capitalize on decentralized city-size heterarchies across Eurasia.

Thompson (2000:91) defines global wars as "only those intensive conflicts that lead to a new phase of significant reconcentration and global political-military and economic leadership." While they are global in scope only after 1500, regional equivalents seem to occur between 1000-1500. The occurrence of phase-shifting wars is indicated in table 3 by single asterisks in the fourth column for K-waves (a double askerisk for 1250 is ours, for the city-state conflicts in which Genoa captures Pisa, thus dominating the maritime trade of the Florentines, and in which Genoa defeats Venice just before 1300).

Carrying the "leading innovation – following structure" argument further, which will require a whole research agenda to complete, what do we see in table 3 in the next period in which corporate innovations of the Venetians and organizational innovations of the Portuguese flourish within a heterarchical Q-period? It is only long after these innovations are operative that the urban hub structure of the trading network shift over from hierarchical to low-q egalitarian. The shift internalizes the corporate and financial organization of innovators into the banking centers, say, of Amsterdam. Similarly, in more self-limiting form, the Church-brotherhood entrepreneurship of the North German Hanse traders in the heterarchical Q3 regime, becomes the corporate form of organization of Hanse trading in Q4 before it is driven out by other forms of competition.

This preliminary comparison and validation-check of q measures and Q-periods, then, leads in a direction where we can begin to reformulate a more dynamical version of our network-economic theory in relation to waves of economic innovation and cycles political leadership. The next stages of research on these issues, which we do not attempt here, will need to bring in and reexamine interactions with internal and external conflict, the role of transport costs on land and on sea, the maritime technology revolutions, the causality of transitions, and the questions about endogamous dynamics in terms of time-lagged interactions, and the role of our network variables. Part of what may be needed in further investigation of dynamics is to study the extent to which innovative practices, if successful, benefit from a contrarian strategy as against the network structure the current Q-period, and lead to the transformation of those structures in the next. This seems to be the gist of Arrighi's (1994) hypothesis that is tentatively borne out by our examination of the innovation and city scaling data, although we do not follow Arrighi's empirical argument in detail.

9 Conclusions

Our exploration of q-exponential scaling of city size distributions led to the discovery of qualitatively different Q-periods for egalitarian (Zipf-like) and hierarchic size distributions in six contrastive 200 ± 20-year periods in the last millennia. Validation checks with a variety of other historical sources established that this variation is a feature of global city networks that is relatively constant for each of the eLW economic Long Waves derived from Modelski's long learning cycles (LLCs) for periods of economic breakthrough. One way to interpret this finding is that the Leading Nation in the second half of a Q-period is replaced by one that sets the new urban economy template or Q-period. The two political phases in an eLW involve, first, an innovative leading sector breakthrough and payoff period for a leading power in regional economy A, followed by a downturn for A while competitor B builds an economic base and then a network for a new innovation that takes them into the next economic eLW cycle.

The K13–K14–K15–K16 (LCC4) series in table 2, for example, represents the peak and fall for the breakthrough to the industrial revolution market economy, but as an LLC does not correspond to a constant Q-periods. Reordering adjacent LLC series starting from assumption of global power after successful execution in global war, as discussed for comparisons of Q-periods and LLCs in table 2, however, matches Q-periods to global political cycles, which are also highly concordant with economic waves.

Oscillations of Q-periods, then, reflect economic network support systems morphing one into the other, but each type supporting a particular political cycle with at least two contending polity leadership phases. Each polity phase supports two leading economic-sector phases. These combine four differentiated K-waves together to form embedded phases that dynamically interconnect in space and time. The basis of the dynamics involves competition: sectoral economic competition and succession, political rivalries affecting sectoral, spatial, and network dominance, and network competition between typical players who are constrained by the network structure and emerging elite players who are slowly reshaping the network structure over time. In investigating this multilevel and global network context of competitive dynamics, the structural variable that is slowest to change is the shape of the global city size distribution.

Cities are dependent on networks of exchanges organized on the basis of sectoral innovations. If power includes the kinetic energies required for transport and the mobilization of goods and labor to support urban systems, wealth is that energy transformed stored as potential, which in turn requires defensive barriers. The markets formed by the continal renewal of these transformations is unstable. The urban network itself and its size or Q-distribution is a property of the action of these markets in the long run, unlike K-waves or L-cycles, which operate at the actor level of local and sectoral economic innovations and the political entities which success builds upon and acts to protect their reservoirs of wealth and exchange potentials. One of the breakthroughs of network research is to begin to delineate the dynamics of interdependence between local action and global structure. The discovery of long-term Q-periods seems to indicate that the interaction between local processes and global structure does not settle into a steady equilibrium. Growth of a heterogeneous ("egalitarian") global city hierarchy, while sustainable for long periods, gives way to conflict, and conflicts eventually precipitate into wars at the largest scale of integration, whether regional or global, operative in a given period. Disruptive conflicts of trade at the broadest level segments trading regions and this segmentation becomes part of the network structure, damping growth but offering certains kinds of new options for the accumulation of wealth. A buyers' orientation towards safety in the segmented accumulation of wealth comes to dominate in this high-q type of Q-period. Like faster-moving market dynamics, this type of Q-period is unstable in the long run, as entrepreneurial innovation begins to interconnect segments in more cohesive ways, an a seller's orientation towards capitalizing on the broader kinetics of exchange through throughputs becomes to dominate. This change, too, seems to require a buildup to global war as a prelude to the establishment of a more global governance of trade. In each case of Q-transformations, one of two types of institutionalized network and urban structure tends to be challenged and eventually overcome by counter-cyclic innovators intent on reaping new benefits from new forms of new limitations on economic interactions.

Doublings of periods are one of the signatures of complex embedded processes. Indeed, each half of an economic long wave (eLW), as shown in table 2, tends to correspond to a leadership cycle that consists of a political rise and fall of the city or nation that is successful in the first part of the eLW but whose economic success is rivaled by another polity that emerges in the second part. Doubling again, the economic success story in the first part of an eLW for a particular polity consists of two cycles or K-waves of innovation, the first pair one of Breakthrough/Payoff, the second one of Base/Network for a subsequent breakthrough. Q-period boundaries, however, do not correspond to simple oscillatory models but have somewhat greater complexity and interactive dependency on such factors as outcomes of global wars. As we showed for the Modelski-Thompson (1966:54) data in our table 2, global wars and their outcomes play a major role in Q-period transitions, especially as they affect subsequent alliances and governance that contributes to the safety of landed trade. The role of maritime protection for trade, naval piracy, and naval strength is also important. While maritime factors are heavily emphasised by Modelski and Thompson, they add complexity to an attempt at modeling. Determining the precise thresholds of quick-to-change Q-periods looks like a promising and soluable problem but requires still further study.

Because the low-q egalitarian thin tailed city size distributions approximate the fixed exponent of the Zipfian distribution that is taken as a norm for urban hierarchies, the intellectual puzzle posed by our research was to understand why urban size hierarchies, for *more than half of the past millenium* should deviate from this supposed norm in alternative periods that have the same stability characteristics as those Q-periods that are more Zipfian. This led us to more careful investigation of the historical processes associated with observed structural changes in the shape of urban size distributions. Our initial thinking about the interpretation of varying Q-periods was that thin tail exponents closer to the Zipfian were the "normal case" for healthy, growing city-systems, and that a more hierarchical thick-tailed city-system, although it occurs worldwide for more than half the millenium, might represent a "disturbed state" of economic depression following major breakdowns and decline in the Eurasian city network, and due to such disruptions as the black death, or the conflicts of the Renaissance period.

The view that high-q periods are representative of long economic depressions is hard to square with the economic and political innovation periods identified by Kondratieff and Schumpeter, and more recently the K-waves and L-cycles of Modelski and Thompson. Economic and political innovations have similar patterns of occurrence in both high and low Q-periods. Attributing "abnormality" to high Q-periods is also hard to sustain when the industrial revolution is considered because the period 1740–1950, while partly one of emiseration of labor, was also a period of a high-q hierarchical city size distributions.[6]

Our development of a network-economic hypothesis about network navigability in high-q urban periods offers an alternative view of the potential for a stable economic equilibrium that would sustain this regime even in the face of land-based conflicts. This hypothesis remains to be investigated empirically and through simulations. The idea is that potential trading benefits are available in "interurban neighborhood" network dynamics give a more hierarchical non-Zipfian urban size distributions. These might offer competitive advantages to the average city in a trading network. This model, initially developed by Adamic and associates (2002), and generalized by White and Houseman (2002) as well as White and Jorgensen (2005), could be tested with a measure r that might relate to the navigability of a network given the steepness of its hierarchy of hubs. Initially applied to measures such as number of links to define the hub hierarchy (hence the literature on scale-free networks as defined by Barabàsi, 2002), we generalize hub navigability here to the probability, given a q-exponential distribution of city sizes acting as the measure of hubs, and varying values of q, as to whether the average city (with log increase in number of direct routes to neighbors) will tend to have a larger city in its neighborhood. If so, then the city network is navigable by moving up the city hierarchy to cities at the highest size levels in an entire region.

[6] Our initial thinking was that this was an exceptional case because the industrial revolution also embodied the diffusive potential for industries to move out into the countryside or smaller towns, with technological improvements offering "lighter" and more transportable industries.

We have not fully answered the question as why low-q periods are equally unstable in the long run as are high-q periods. One possible intuition might be that greater navigability implies greater fairness through price equilibration because the size differences in a navigable trading route are relatively small when hubs are thin-tailed. When potential navigability ceases to be a property of an exchange network, smaller cities in a city network are at a trading disadvantage relative to the great urban hubs. These thin-tail hubs necessarily reach out to smaller cities by more long-distance, *inegalitarian* hub-centered networks, and the distance and differential of their long reach emphasizes trading differentials and price disadvantages. The hypothesis that might be tested here is that egalitarian Q-periods, as opposed to navigable Q-heterarchies, entail significantly more unfair rural/urban terms of trade. An alternative hypothesis is that low-q periods are more likely to produce inflation because of faster population growth, thus greater economic inequality, leading to long-run instability through conflicts internal to polities. This is not a systemic feature of a Q-period network, however, so it would not explain a macro shift in Q-period unless problems of excess population pressure on resources were somehow synchronically entained across the network.

City network navigability still needs to be studied through simulation, but is consistent with what we observe during the industrial revolution and other periods of heterarchic size distributions: the outsourcing of production from larger to smaller cities. With this lens, we also see this happening in northern Europe after 1290 when conflicts disrupt along the land routes for trade between Italy and the Low Countries, as the city size distribution shifts to heterarchic and industrial startups and eventually banking networks sprout up throughout the decentralized Holy Roman Empire of the German line of kings. We can also see, as opposed to an eternally locked-in urban inegalitarian hierarchy, the possibility of contemporary oursourcing recreating the heterarchies typical of these earlier historical periods.

Modelski (personal communication) reports on our funding as follows: "World cities occupy roles that extend beyond the global economy, they a hubs of a plethora of social networks, from family, ethnic, to cultural, political, centers of migratory currents and the flow of experts. That is why the significance of the new insight might be broadened. One constitutive process of world system evolution is global community formation (period of 1000 years, Devezas-Modelski 2003:836). World cities are i.a. major foci of social organization, the infrastructure of globalization, and the building blocks of (emerging) global community. We might therefore argue that the first four of the modern alternations of the Q-shapes be viewed as four phases of one period of global community formation. The swings to (hierarchical) concentration in time find their limit, and move (twice) toward a flatter landscape. The four Q periods might be seen in this way, as concentration in the Song empire, followed by Mongols ravages (de-urbanization), and opening of Eurasia, resuming (in a decisive fashion) in monopolistic maritime organization, followed ultimately by more recent dispersal of urban power."

"Then there is the 'anomaly' of the current (5th) phase of that process. Rather than being a mere repetition it would seem that it marks the start of a new period of community formation (as well as the evolution of the global economy) characterized by high population growth as well as the rise of very large cities in a manner totally unprecedented in the history of urbanization - in the space of one century, between 1900 and 2000, the number of millionaire cities rose from 16 to 363! (and that is for cities proper only) creating a strong and hierarchical concentration of urban power that is also laying down a base for democratization (urbanization favors democracy) that will take another century or so to complete, creating in its turn a flatter system. This is no mere alternation between two equilibria but a breakthrough to a new form."

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