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Historical Population Dynamics:
A Model of Pre-Industrial Demographic Cycle

Introduction

We believe that one of the most important recent findings in the study of the long-term dynamic social processes was the discovery of the demographic cycles as a basic feature of complex agrarian systems' dynamics.

The presence of demographic cycles in the pre-modern history of Europe and China has been known since quite a long time (e.g., Postan 1950, 1973; Abel 1974, 1980; Le Roy Ladurie 1974; Hodder 1978; Braudel 1973; Chao 1986; Cameron 1989; Goldstone 1991; Kul'pin 1990; Mugruzin 1994 etc.), and already in 1980s more or less developed mathematical models of demographic cycles started to be produced (first of all for Chinese "dynastic cycles") (Usher 1989). At the moment we have a very considerable number of such models (Chu and Lee 1994; Malkov and Sergeev 2001, 2002, 2004; Malkov *et al.* 2002; Malkov 2002; Turchin 2003; Nefedov 2002a; 2004).[1] We would like to discuss in some detail three approaches to modeling of demographic cycles: Turchin's (2003) models, another by Chu and Lee (1994), and finally, the model of Nefedov (2002a; 2004).

The "demographic-fiscal" model developed by Turchin (2003:118–27, 208–13) connects population dynamics, state resources and internal warfare. In this model the elites controlling the state are not assumed to be selfish. It is rather assumed "that the state has a positive effect on population dynamics; specifically, it increases *K* [the carrying capacity]" (Turchin 2003:122). "There are many mechanisms by which the state can increase the carrying capcity… The strong state protects the productive population from external and internal (banditry, civil war) threats, and thus allows the whole cultivable area to be put into production… The second general

[1] There are also a rather large number of pre-industrial population dynamic models designed to account for "the escape from Malthusian Trap", rather than for the structure of pre-industrial population cycles (Artzrouni and Komlos 1985; Steinmann and Komlos 1988; Komlos and Artzrouni 1990; Steinmann, Prskawetz, and Feichtinger 1998; Wood 1998; Kögel and Prskawetz 2001; Komlos and Nefedov 2002).

mechanism is that states often invest in increasing agricultural productivity by constructing irrigation canals and roads, by implementing flood control measures, by clearing land from forests, etc. Again, the end result of these measures is an increase in the number of people that can be gainfully employed growing food, i.e., the carrying capacity" (120–121). Thus the depletion of state resources and state breakdown are assumed to be leading to the decline of the carrying capacity and, thus, demographic collapse. As in all the other demographic cycle models the per capita rate of surplus production is assumed to be a declining function of population numbers, whereas the state expenditures are assumed to be proportional to population size. Within this model "the rate of change of S [state resources] is determined by the balance of two opposing forces: revenues and expenditures. When N [population] is low, increasing it results in greater revenues (more workers means more taxes). The growth in state expenditures lags behind the revenues, and the state's surplus accumulates. As N increases, however, the growth in revenues ceases, and actually begins to decline. This is a result of diminishing returns on agricultural labor. However, the expenditures continue to mount. At population density $N = N_{crit}$, the revenues and expenditures become (briefly) balanced. Unfortunately, population growth continues toward the carrying capacity, K, and the gap between the state's expenditures and revenues rapidly becomes catastrophic. As a result, the state quickly spends any resources that have been accumulated during better times. When S becomes zero, the state is unable to pay the army, the bureaucrats, and maintain infrastructure: the state collapses"[2], which leads to a radical decline of the carrying capacity of land and demographic collapse (Turchin 2003:123).

Turchin has also developed a number of elegant models of population dynamics, where the peasant-elite interaction plays the role of the main mechanism of state breakdown. When the population size becomes large, food supplies are exhausted and the elite multiplies out of control – then state collapse is observed, followed by a significant decrease in the number of peasants. A large number of elite cannot be kept up by a shrunken population, so eventually the elite decreases, and the cycle of growth starts over. A resulting feature in Turchin's model is that we do not observe the population to climb up to its carrying capacity and saturate at a certain level before a collapse. Also, in this model by Turchin the elite behaves in a strictly selfish manner; it does not play a role in food redistribution (e.g. to provide food for starving people during time of famine).

[2] We would rather say that this leads to attacks on the state by dissidents of various sorts and this warfare leads to state collapse.

The interesting model of Chu and Lee combines elements of mathematical modeling and statistical analysis/best fit approach. The main idea is very attractive. The population consists of rulers, peasants and bandits (rulers being equated with soldiers, drafted every year at a constant rate). The population has some intrinsic growth rate, that is, the rate at which it increases given unlimited resources. As the density increases, the resources get scarce, and the growth rate decreases (this is an effect of overpopulation). At the same time, there is a flux of people from peasants to bandits and *vice versa*. Each person faces a choice of either working in the field or "defecting" and getting his food by means of force. The soldiers are supported by taxation and they fight the bandits. The rational choice is based on evaluating the "utility function" of peasants and bandits and it depends on external circumstances such as the degree to which agricultural resources are damaged by warfare. The utility is a function of the food share received and the probability to survive.

As the density of the population grows, it becomes more and more likely that people choose to become bandits and fight for their food instead of growing it. This leads to the reduction in population numbers and the cycle starts over.[3]

Chu and Lee did not specify their model to the extent where it can be implemented directly. Instead, they used it as a tool to improve the fitting of real historical data. Information on warfare and winter temperatures was included as exogenous variables, and the frequency of peasant rebellions was modeled based on the expected fraction of the rebels, calculated by the model. This gave an excellent fit to the existing data.

Another interesting idea presented by Chu and Li is the two possible explanations of the irregularity of the historical demographic cycle. One explanation is simply the ("external") stochasticity of the climatic conditions. The other one is the intrinsic chaotic behavior of the dynamics system. Depending on parameter values, the simplified system analyzed by the authors has been shown to undergo a series of period-doubling bifurcations and a transition to chaos.

What is slightly discouraging is that the authors did not include the effects of a yearly changing crop yield (which is a function of climate fluctuations). Nor did they include the positive role of the ruling class in food redistribution (which is especially salient precisely in the Chinese case on which their model is based). Moreover, the historical temperature data are shown to be irrelevant for the fit. These points will be addressed in more detail when we talk about our model.

Nefedov, who incorporated stochastic effects of year-to-year food yields on the population dynamics, has found another approach. He noticed that as the population reaches the carrying capacity of land, and food storages become depleted, then random effects of good and bad years can play a significant role in the dynamics. As food becomes very scarce because of, say, a bad winter, people tend to sell their land and leave for cities, or join bands of rebels. In idealized conditions, that is, given a perfectly constant food yield, no cycle is expected. However, a bad harvest triggers a mechanism of collapse with a significant reduction in population number. Nefedov's models have a lot of interesting components. For example, because of the increasing numbers of people leaving land as population density increases, we expect to see an intense growth of cities, which is confirmed by historical observations. What seems to be missing from Nefedov's models is the direct role of rebellion and internal warfare on the cycle behavior. If only economic factors are taken into account, then there seems to be no inertia in the dynamics, and each demographic catastrophe is followed immediately by a new rise. This plainly contradicts historical data where "intercycle" periods of variable (but always significant) length are observed.[4]

It is not very often that we have direct evidence for long-term trends for both population numbers and consumption rates. It is very rare that we have long-term data on both variable dynamics within a cycle. We have practically no long-term population data outside China (and, to some extent, Europe), and this made it difficult to detect demographic cycles outside Europe and China. However, not so infrequently we can find long-term data on some other variables whose dynamics is predicted by Nefedov's model (first of all per capita consumption rates), and quite regularly they have the form just predicted by Nefedov's model**.** Using such indirect data, as well as his system of qualitative indicators of various phases of demographic cycles Nefedov (1999a, 1999b, 1999c, 1999d, 2000, 2001a, 2001b, 2002a, 2002b, 2003, 2004 etc.) has managed to detect more than 40 demographic cycles in the history of various ancient and medieval societies of Eurasia and North Africa, thus demonstrating that the demographic cycles are not specific for Chinese and European history only, but should be regarded as a general feature of complex agrarian system dynamics.

[3] The term "bandit" in Chinese is ambiguous and includes landless migrants who disturb the social peace. Hence, they are problematic because they are hungry and have no legal rights in the villages where they reside.
[4] See Korotayev's article "Historical Population Dynamics in China: Some Observations" in this issue.

Mathematical Model

The focus of the present work is not to fit the existing historical data, with all the intricate features. Therefore, we do not feed in the model external warfare or temperature variables. Instead, we would like to mimic the qualitative behavior of the system in order to see whether the historical dynamics is consistent with verbal explanations offered by various authors. We would like to understand in what way annual climatic variations interact with population density to produce a demographic collapse through an increased frequency of internal warfare. We would also like to see how the country gets out of the state of disarray, and what factors influence this "intercycle" period. Internal warfare and its inertia will play an important part in this model.

The only exogenous variable in our model is a fluctuating climate (that is, a fluctuating harvest yield). We will run our model and observe how cycles are formed, and what influences their period and amplitude. We set up the model as a set of difference equations where the value of the variable in a certain year is defined by the state of the system in the previous year.

We denote by N_i the number of agricultural households in year i. Let us suppose that the total area of the land available for agriculture, is A_{total}, and the area per household is $Area_i$. In times of peace, the amount of land per household is roughly $Area_i = A_{total} / N_i$, that is, all available land is being used.

Let us denote by H_0 the average amount of food harvested by a household each year, measured per unit area. Every year, due to changing weather factors, the harvest yields will be different. We model this by setting the harvest variable to be $Harvest = H_0 + random\ fluctuations$. The amount of food per household per year is then given by $Food = Harvest * Area_i - seed$, that is, it is the total amount of food collected minus what has to be saved up to seed the fields next year.

There is a minimum amount of food needed for a household to survive each year; we call it $Food_{min}$. Then the quantity $dF = Food - Food_{min}$ is the food surplus. In a good harvest year, this quantity is positive, in a bad year it is negative (food shortage). The population grows, or shrinks, depending on this factor. Namely, the amount of population growth per year is directly proportional to dF, and if $dF > 0$, then the population grows, and it shrinks otherwise. This is captured by the following basic growth model,

$N_{i+1} = N_i (1 + \alpha\, dF)$,

where α is a proportionality coefficient; we restrict the growth rate, $\alpha\, dF$, by the maximum of 2%. This model implicitly includes the carrying capacity of the land, in the following way. If there is a lot of available land, then the peasants will have large allotments and will collect enough food to feed their families, even in difficult years. As a result, the food surplus will be positive, and the population will grow. New households will need land, and therefore area per household will become lower (the total area of land available for agriculture is fixed). The food surplus will be lower, and the growth will slow down, until the system reaches a “dynamic equilibrium”. This is a typical Malthusian growth model, leading to a “logistic” growth curve with saturation.

In reality, things are more complicated, and the first factor that we take into account is the presence of the state. We assume that the state collects taxes, whose amount per household (in years of peace) is determined on the basis of food surplus. We assume the following taxation scheme: If there is additional product, the state collects a fixed fraction of food surplus. If there is shortage of food, the state does not collect taxes. A state-owned food storage is formed, which is then used to feed the starving people in the years of poor yield. We keep track of the state-owned food storage by means of the variable S_i. If the food surplus is positive, there is an increase in the food storage by the amount of $N_i\ tax\ dF$, where *tax* is the proportion of the food surplus collected as *taxes/rents.* If there is shortage of food, then the stored food is distributed among the peasants, and the amount of stored food decreases. It can be completely depleted in some bad years. If we include the food storage in the model, we can see that in the beginning, as the population grows, the food storage increases, and when the population reaches saturation at the carrying capacity of land, the food storage gets depleted after a few bad years, and keeps oscillating at low levels.

Next, we need to add some political factors to this purely economical model. In time of trouble, some peasants may lose land through processes of indebtedness and seek to obtain food by becoming itinerate farm workers or by joining a band. As the supply of food declines per capita, increasing numbers of peasants will be forced off of their land, becoming “bandits”. Thus we introduce the variable, R_i, the number of bandits in year i. The number of peasants becoming bandits each year is set to be $dR_i = -\alpha_{out} N_i\, dF/Food_{min}$ in the years when there is food shortage, and 0 in prosperous years. The number of peasants who will be forced off of the land for a given dF depends on the distribution among peasants of the shortfall. This shortfall will never be the same for all peasants. This distribution of shortfall will affect the percentage of peasants who become chronically in debt and lose land. Hence, we introduce α_{out} as a random variable which is a function of the annually-variable distribution of the shortfall.

In order to describe the population dynamics of the bandits, let us suppose that the bandits survive by extracting resources from peasants. It is easier for bandits to survive when the peasant/bandit ratio is larger. Let us introduce the quantity, $\delta_{i,}$, equal to $\delta_i = 1 - N_i / (10 R_i)$ if $N_i < 10 R_i$, and $\delta_i = 0$ otherwise. Then the equation for the number of bandits each year can be written as

$$R_{i+1}=R_i (1-\beta - \delta_{i,}) + dR_i$$

This equation states that the population of bandits decreases in the absence of new recruits. Bandits have high death rates due to inpredictable incomes and hunger. The death rate of bandits consists of two parts, β is the constant background rate, and δ_i depends on how successful the food extraction proceeds from peasants. If the peasant – bandit ratio is greater than 10, we assume that their death rates are no greater than those of other peasants . The smaller is the ratio, the harder it is for the bandits to survive.

As the next step, we need to discuss the impact of banditry on the lives of peasants and the general condition of the state. First of all, the presence of bandits ravaging through the country introduces a certain “fear factor”. If the population of the country decreases due to the intensification of the internal warfare, a lot of land becomes free, and in principle could be cultivated by the remaining peasants. However, this is not likely to happen as peasants tend to stay in protected areas, and agricultural activities outside the limited protected region are considered unsafe. This can be modeled by assuming that in the presence of bandits, some parts of unoccupied land are not available for peasants: $Area_{i+1} = A_{total} / (N_i +10 R_i)$. This means that in times of peace ($10 R_i << N_i$), all available area is distributed among the peasants. In times of war ($10 R_i \sim N_i$), the peasants tend to stick to their land, and expansion to available lands does not happen, and effective carrying capacity of land decreases. Thus in our model, banditry/rebellions/internal warfare are the basic reasons for a demographic collapse, as described in our model.

Secondly, the warfare makes its negative impact on the ability of the state to collect taxes. In order to capture this, we introduce the “internal warfare coefficient”, U. Depending on the level of internal warfare (measured best by the number of bandits), this coefficient is equal to 0 in times of war, and it is equal to 1 in times of peace. We assume that during internal war, the state is weakened and its ability to collect taxes is impaired. This is captured by the following equation:

$$S_{i+1} = S_i + dS * U,$$

where dS is the tax collected or the food from the storage given back to the peasants, exactly as described before, and U quantifies the efficiency of state and the impact of internal war on tax collection. In the time of internal war, U is close to zero, and taxes are not collected efficiently, because the state infrastructure is partially destroyed.

To summarize, we have included three main elements in this model. (i) The Malthusian-type economic model with elements of state as a tax collector, and direct impact of yearly harvest yields, describes the logistic shape of population growth. It explains well the upward curve in the demographic cycle and the saturation as the carrying capacity of land is reached. (ii) Banditry and the rise of internal warfare in time of need are the main mechanism of demographic collapse. Personal decisions of peasants to leave their land and become warriors/bandits/rebels are influenced by the economic factors. (iii) The inertia of warfare, which manifests itself in the fear factor and the destruction of infrastructures, is responsible for a slow initial growth and the phenomenon of the “intercycle”.

Let us put all the definitions and equations together in a coherent set of difference equations, describing demographic cycles of complex agrarian systems

N_i is the number of peasants
R_i is the number of bandits
S_i is the amount of food storage

$$Harvest = H_0 + random\ fluctuations$$

U is "internal warfare inertia coefficient"

$$U = \begin{cases} [1 - R_i\ /\ (0.03\ N_i)]^{\wedge}3 & \text{if } R_i < 0.03\ N_i \\ 0 & \text{otherwise} \end{cases}$$

$Area_{i+1} = A_{total} / (N_i +10 R_i)$

$Food = Harvest*Area_i - seed$

dF is "food surplus" (which could be not only positive, but also negative)

$dF = Food - Food_{min}$

$dS = N_i\ tax\ dF$ if there is additional product

$N_i\ dF$ if there is shortage of food and enough storage

$- S_i$ if there is considerable shortage of food.

dF' is "effective" food surplus, after tax/rent payment, or after receiving state help

$dF' = dF - dS / N_i$

dR_i is number peasants turned into bandits in year i

$dR_I = -N_i\ dF / Food_{min}\ \alpha_{out}$ if there is not enough food

0 if there is enough food

α_{out} is "peasant-bandit transformation rate"

$\delta_i = 1 - N_i / (10 R_i)$ $if\ N_i < 10 R_i ,$

0 otherwise

α is related to the growth rate of peasants. We assume that the quantity $\alpha\ dF'$ cannot exceed 0.02.

$\beta + \delta_I$ is death rate of bandits

rob – bandit-related death rate of peasants

The difference equations (equipped with appropriate initial conditions), are:

$S_{i+1}=S_i + dS\ U$

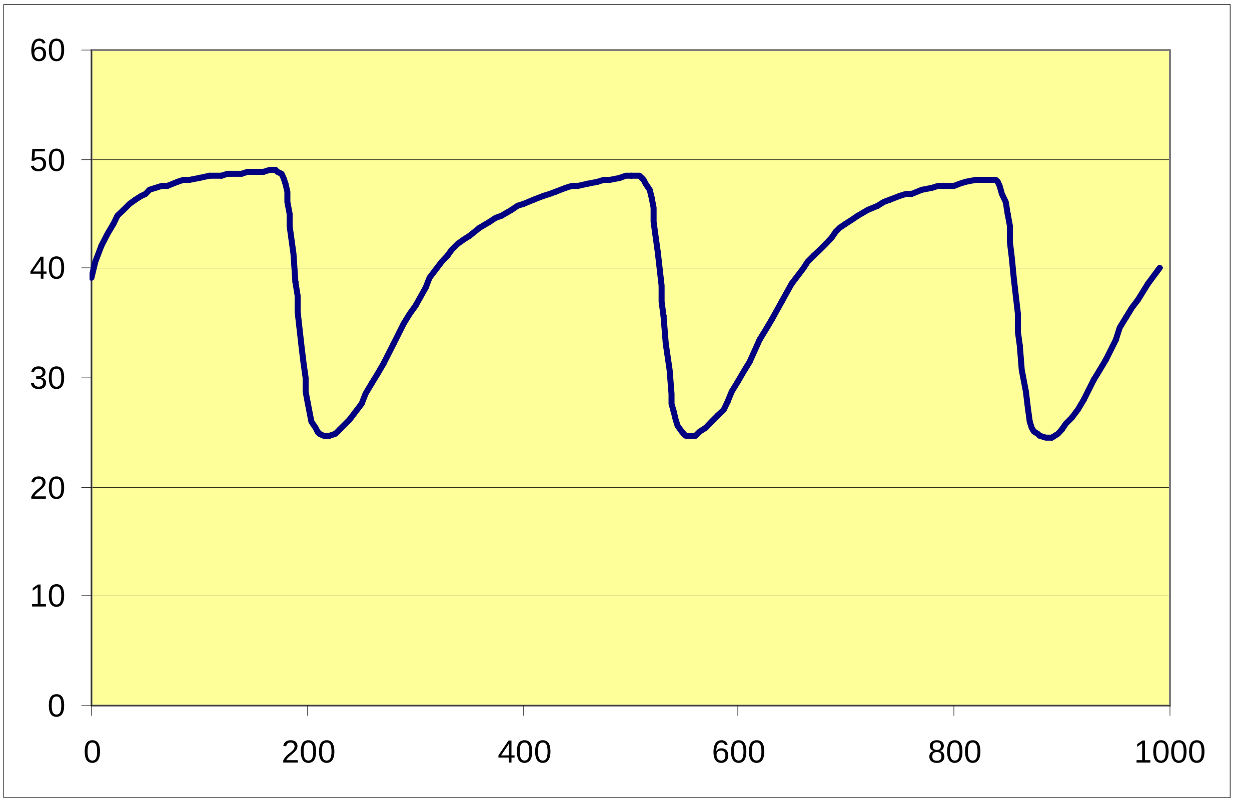
$N_{i+1}=N_i (1 + \alpha\ dF') - dR_i - rob\ N_i R_i$

$R_{i+1}=R_i*(1 - \beta - \delta_i) + dR_i$

NOTES: We follow Chu and Lee (1994) in denoting as "bandits" both bandits and rebels (note that Chu and Lee in their turn just follow the Chinese tradition to denote both categories with one term [*fĕizéi*]). The logic behind this merging is that both groups produce rather similar effects on population dynamics: rebels might even fight with aim to improve the life of peasants, however, in order to feed themselves they would still have to take food from peasants. The presence of the state-sponsored relief system may seem to be too China-specific. Indeed, any developed systems of this kind are very rarely found within complex agrarian systems outside East Asia. However, in fact, some famine-relief counter-crisis sub-systems are found within an overwhelming majority of complex preindustrial states. Indeed, developed state-sponsored systems were found quite rarely; however, almost all complex agrarian states still possessed some counter-crisis subsystems. The most wide-spread type was based on the private stores of food resources held by elites (landowners etc.). During famines the elites would tend to use those storages to provide a sort of relief to at least some categories of commoner population. For example, landowners would not be normally interested in their tenants dying out and would provide some support to them in such cases. Such relief was very rarely quite altruistic. Thus landowners would provide some food to peasants in order to indenture them and to get their land; however, this would still help substantial parts of affected population to survive through lean years. For the sake of simplicity in our model both main types of pre-industrial counter-crisis relief subsystems are merged in one mechanism (hence, taxes are merged with rents).

This model generates the following dynamics (see Fig. 1):

Fig. 1. Populations Dynamics Shape Generated by the Model



It shows a rather close fit with the observed pre-industrial demographic cycle pattern.[5]

Conclusions and Discussion

The proposed model combines positive aspects of earlier models that have not been earlier combined within one model. Unlike Nefedov's model, but like some prey-predator logic based models it accounts for significant intercycle periods. On the other hand, unlike the latter, through inclusion of the famine relief subsystem dynamics into the model it accounts for lengthy periods of very slow and unsteady population growth (when most part of the population had inadequate per capita acreage and there existed very strong incentives to innovate in the raising of the carrying capacity of land). Hence, this model could be used as a basis for development of a new generation of models accounting both for "secular cycles" and "millennial trends". In order to do this, we suggest in future to alter basic assumptions of the earlier generations of demographic

[5] See Korotayev's article " Historical Population Dynamics in China: Some Observations" in this issue.

cycle models (that both the carrying capacity of land and the polity size are constants). These are variables with long-term trend dynamics in the rise of carrying capacity of land, cultural complexity, and polity sizes that the new generation of models needs to account for. Demographic cycle models account at present only for cyclical dynamics; the new modeling should be able to interconnect trend with phase dynamics.

There are both theoretical and empirical grounds to maintain that the carrying capacity of land not only experienced an upward long-term trend, but also that the innovations contributing to this trend occurred in particular phases of demographic-political cycles. Incentives created by relative abundance of resources in the initial upward growth phases of agrarian state demographic-political cycles tend to be insufficient to create the innovations leading to the rise of the carrying capacity of land (these phases, however, were also very important, as during them there were strong incentives for the innovations leading to the rise of productivity of labor). Rather, innovations raising the carrying capacity of land tended to occur during intermediate periods before the demographic collapse phase. While these innovations usually acted only to delay demographic collapses, they secured the existence of a very important upward trend, which could be to some extent accounted for as a by-product of the demographic cycles.

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