

**Re: A new approach to social dynamics (02-18-2004)**

**by Don Saari:**

<http://eclectic.ss.uci.edu/ResFocusGrp/talks/SaariLecture2-18-04.pdf>  
Presentation for the Social Dynamics and Evolution group of MBS

How can we understand historical process?

**Response commentary and questions**

**by Doug White**

Revised 2-27-04

RE the talk, point 3. Modeling dynamics in the social sciences (see notes):

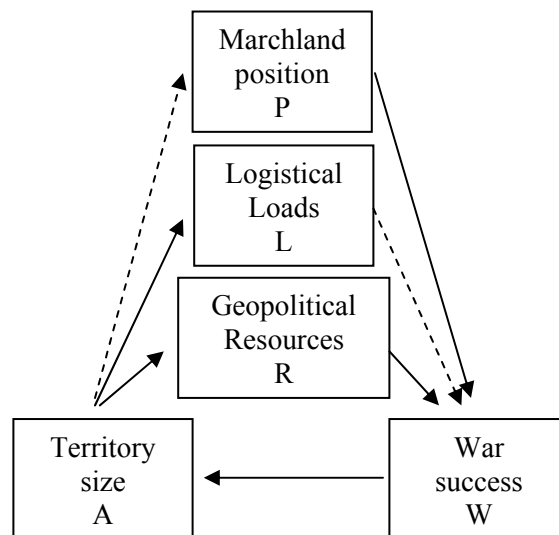
e.g., Turchin, *Historical Dynamics* 2003:

He has models of Area expansion/contraction such that change in A,  $\Delta A$  or  $\dot{A} = f(x_i)$ , where are the variables that influence A.  $\dot{A} = f(x_i)$  is a correct beginning. He makes an error, however, with such equations as that on p. 17, which also implies a threshold between exponential growth and decline.

$$\dot{A} = c(A - d) \quad (\text{Turchin's 2.10})$$

This assumes the conclusion in order to prove the conclusion. This is a standard social science error: assume what you want to get and get a nice exponential equation. Then to believe that the nice equal describes what we are looking at. But these assumptions already contain the conclusions. It is not surprising that you get the results that you have assumed. We should take the attitude "I don't believe it" unless the equation accurately describes a body of data we are looking at, as in experimental data. The same errors occur in evolutionary biology or the study of party systems in political science.

True, this is a model of exponential growth where c and d may be constants for a given case. Saari then assumes that Turchin is talking about one single case. Not so fast! This is far too facile. Equation 2.10 derives from  $(\Delta) \dot{A} = rR + wW$  as part of a critique of Randall Collins' model diagrammed (p 16) as follows, and refers to only the A-R-W feedback loop. As we shall see, Turchin rejects this model for the same reasons as Saari.



Solid/dashed arrows for positive/negative effects

**Figure 2.2. Feedback structure of the Collins geopolitical model.**

Let's get clear on the model, since Bell and Sandeshika Sharma are proposing further study of the multiple loops in Figure 2.2, ones that Turchin also rejects. First, the  $cA = rR$  in  $\dot{A} = c(A - d)$  represent the (mostly) human resources of A, such as available recruits for defense and taxes paid by the population. Second, the c in  $rR=cA$  translates A into geopolitical power through a constant of proportionality r. Third term  $cd = wW$  represents warfare success W ties a constant. Change in A is not a simple function of itself. The difference here is between Saari's assumption that c and d represent constants for a given case, while for Turchin they vary across cases, and while  $\dot{A} = f(A,W,R)$  for different cases, Collins' model ought to provide dynamics for changes in W and R for individual case, so that the equation  $\dot{A} = c(A - d)$  is invalid from the start. Saari is not seeing Turchin as a fellow dynamicist. Beware a too-quick interpretation of what a formula refers to.

The whole of Turchin's chapter 2 is in fact a criticism of existing models of historical dynamics. Figure 2.2 is not what Turchin is proposing. In the chapter, also written as a primer on dynamics, he distinguishes between (1) zero-order dynamics that arise in systems that are not affected by negative feedbacks, such as boundless growth (e.g., 2.10) or decline, (2) first-order dynamics that generate stable or metastable (one or more) equilibria on a fast scale, such as asymptotic and logistic growth, and (3) second-order dynamics in which feedbacks act with a lag, e.g., a boom/bust dynamic or sustained periodic or chaotic oscillations.

Part of Turchin's critique of the Collins model is that the A-R-W positive loop is a zero-order dynamic with a threshold for unlimited growth versus decline, while adding A-L-W to include logistic loads (L) gives only first-order dynamic, metastability, with a threshold for growth versus decline but also an upper limit to growth. These models do not show any second-order oscillations nor any long periods of decline from a unit that has previously grown large such as we observe in the historical observations. Extensive observational data coded by Rein Taagepera data is used to show, for example, long periods of decline in empires. Nor do the spatial simulation models or the conflict legitimacy models that he reviews. He emphatically rejects all these models. So: Saari, Sharma, Bell, please read the text at least enough to understand how a given model is defined. It is not sufficient to trash someone's book simply because of the algebraic form of an equation taken out of context. **Let's take the mathematics seriously!**

Chapter 4 has the beginnings of Turchin's theory of territorial dynamics, to which I would hope we would pay more attention. E.g., p. 64 (4.1), he again critiques a more elaborated version of (2.10) where h is a spatial scale variable that translates A into geopolitical power and a (assumed to be a constant) is the geopolitical pressure from the hinterland.

$$(\text{delta}) \dot{A} = R(1 - A/h) - a \tag{4.1}$$

Turchin's argument expands on  $R = cA$  to explain that the variable c which translates the potential human population resources in A into actual resources needs to be measured by

the level of collective solidarity (asabíya)  $S$  that generates collective action (willingness e.g., to pay taxes and defend the polity). So 4.1 becomes

$$\dot{A} = c_0AS(1 - A/h) - a \quad (4.2)$$

Note now that  $\dot{A} = f(A, A^2, \text{etc.})$ . Further specification is then given for the temporal evolution of  $S$ .

At this point I want to stop to query or critique one of the ideas advanced by Saari, perhaps overgeneralized:

–is it illegitimate for any  $\dot{A}=f()$  to include  $\dot{A} = f(A, A^2, \text{etc.})$  modeling dynamics?

I should hope not. The standard formula for population growth, for example, is  $\Delta P=kP$  for exponential growth, with variable  $k$  and surely  $\Delta P=f(P,k,P^b, \text{etc.})$  are legitimate ways to model empirical population growth and policy.

In any case, I would hope that Turchin’s modeling efforts would be taken seriously and not rejected out of hand.

I am interested in this example (4.2) not because  $S$  is an immeasurable stand-in for capacity to generate collective action but because in empirical network studies I use an actual measure of group cohesion that derives from the graph-theoretic notion of  $k$ -connectedness, call it  $K$ . I would be interested in testing models where the endogenous process of territorial dynamics, i.e., for a single case, and no interaction with other polities

$$\dot{A} = c_0AK(1 - A/h) - a \quad (4.2)'$$

Let’s follow Turchin a bit further in chapter 4 (64-65):

“I need now to derive an equation for the temporal evolution of  $S$  [DRW substitutes  $K$  from network theory]. The first step is to choose a law of [endogenous] growth for it. We still lack a well-developed theory that would connect micro-level individual actions to macro-level dynamics of [ $K$  as a measure of  $C$ , the capacity to generate collective action, which Turchin postulates as  $S$ ], so the next-best approach is to select among the simple models of growth [linear, exponential, asymptotic, and logistic growth, contrasted with boom/bust or sustained oscillations as simple dynamical models]. The real choices are either the asymptotic or the logistic growth. The asymptotic growth implies that when [ $C$ ] is at low level (near zero), it will grow linearly [actually for network evolution of  $K$  this is not a bad assumption]. This is also not a good assumption. . . . [M]ost people are conditional altruists (if they are altruists at all). When they are not in a minority, conditional altruists are likely to follow selfish strategies, because they do not wish to be taken advantage of by free-riders. It is only when there are a sufficient number of other (conditional or unconditional) altruists around that they will be likely to behave altruistically. This (admittedly crude) argument suggests that the initial growth of [ $C$ ] should be autocatalytic [exponential in early growth]. . . . [I]f we wish to select one of the four

simple models of growth, then it is the logistic that matches best the hypothesized dynamics of [C] growth:

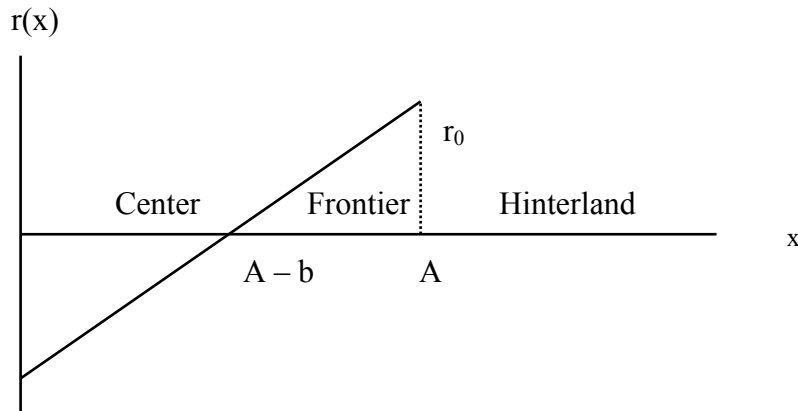
$$\Delta C = r(A)C(1 - C) \quad (4.3)$$

“Note that I have already made the relative growth rate  $r$  a function of  $A$ . Figuring out the form of this function is our next task.

“In the verbal theory advanced earlier in this chapter, I argued that [C] is expected to grow in the locations near the imperial boundary and decline in the empire’s center. Let us assume, for simplicity, that the relative rate of growth of [C] is a linear function of distance from the border (Figure 4.1). We see that the space is divided into three areas: the “hinterland” outside the empire, the “frontier” where *asabiya* tends to increase, and the “center” where [C] tends to decrease. The parameter  $r_0$  is the maximum relative growth rate [of C], which takes place right at the border ( $x = A$ ) and  $b$  is the width of the frontier region (where  $r(x) > 0$ ).

“The average polity-wide [C] is

$$r(A) = (1/A) \int_0^A r(x) dx = r_0 (1 - A/2b) \quad (4.4)$$



**Figure 4.1** The relative growth rate of [C],  $r(x)$ , in relation to the imperial boundary. Symbols:  $A$  is the polity size,  $b$  is the width of the frontier zone (thus the “Center” extends from  $x = 0$  to  $x = A - b$ , while the frontier is between  $A - b$  and  $A$ , and  $r_0$  is the maximum relative growth rate of [C], taking place at the boundary  $x = A$ .

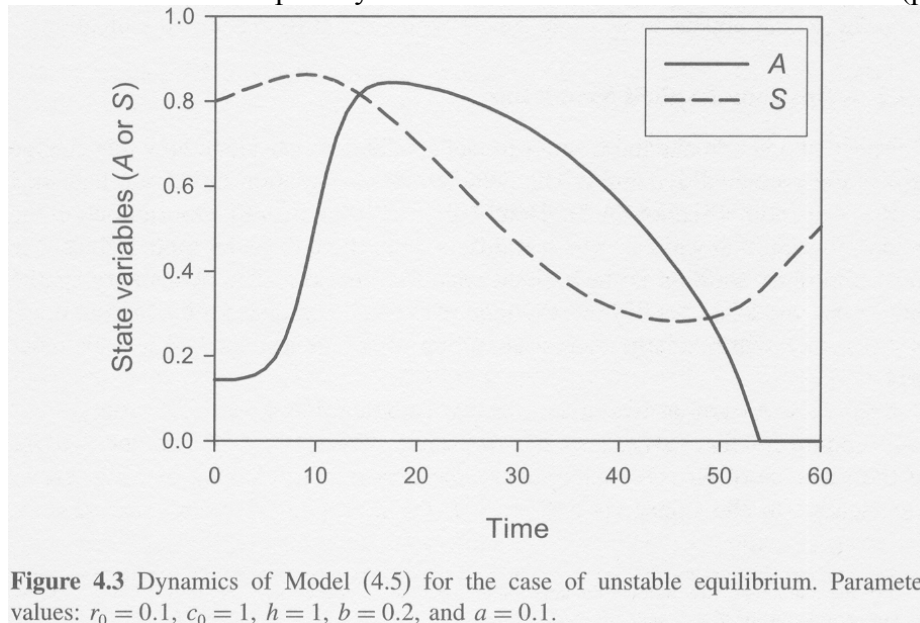
...  
 “Substituting this formula into the equation for  $\Delta C$  ... [ and solving ]

$$\begin{aligned} \dot{A} &= \Delta A = AC (1 - A/h) - a \\ \Delta C &= r_0 (1 - A/2b) C (1 - C) \end{aligned} \quad (4.5)$$

“The state variables are constrained as follows:  $0 < S < 1$  and  $A > 0$ .”

[end of quote] From this Turchin derives an unstable equilibria at  $\Delta C=0$  where  $x = 2b$  is the size of A and an unstable equilibria at  $\Delta A=0$  where  $x = h/2$ .

The results of all this are complex dynamics for A and C as a function of time (p. 67):



**These, then are the dynamics where we need commentary on Turchin’s work, not those of Collins or the models of Chapter 2.**

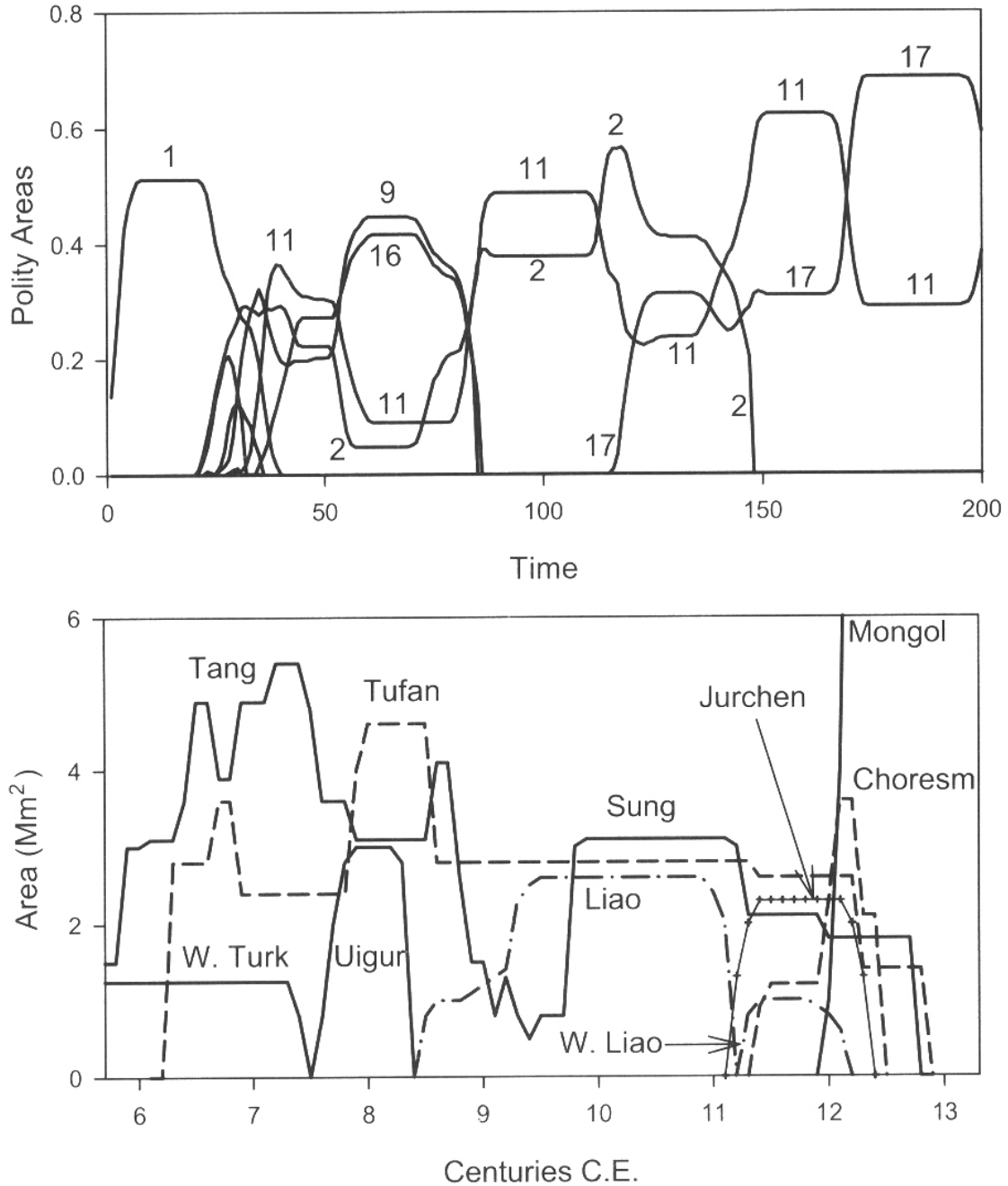
The nonlinearities in these dynamics lead to the following simulated (as compared to actual) results as in Figure 4.4. Here, it would be useful to have commentary:

- A. Turchin has a great deal of data from many cases.
- B. He is concerned with disconfirmation.
- C. He is not “fitting” parameters from data to derive models
- D. He is also using simulation results.

So, let’s take the math seriously. What is he doing? What do these simulations tell us? Might they tell us (winding numbers and all) whether C or K, the latter an unmeasured but measurable variable, might be worth the effort of extensive measurement across many cases?

At this point what Turchin is doing is checking whether the shapes of the trajectories of variable A in the cases generated by simulation have the same properties as the actual trajectories. The observed properties might be thus:

1. relative to maximum growth, initial growth to a local stability is rapid and linear
2. length of local stability relative to initial climb is quite variable
3. from local stability, new growth or decline phase is ~equiprobable



**Figure 4.4** (a) Dynamics of the spatial asabiya-area model. Each curve depicts the territorial dynamics of simulated empires (polity area is expressed as a fraction of the total arena occupied). Model parameters are  $r_0 = 0.2$ ,  $\delta = 0.1$ ,  $h = 2$ ,  $S_{crit} = 0.003$ , and  $\Delta_p = 0.1$ . Numbers associated with the trajectories are the imperial indices of the polities. (b) Expansion-contraction curves of areas for polities in East and Central Asia, 600–1200 C.E. (Data from Taagepera 1997: Appendix)

At this point the research question might be to investigate the five variables of the model. Do the results have the same “shape characteristics” (for want of a better word) :

1. if b is eliminated or permuted? How does frontier size affect the shapes?

2. ditto for a, geopolitical pressure from the hinterland
3. same for  $r_0$ , the maximum relative growth of C
4. and h, the spatial scale variable that translates A into geopolitical power

Note that these four variables are measurable for a given sample (hopefully large) of cases.

Now, however, we have an interesting empirical question: for the sample, only  $r_0$  takes a single value (the maximum) for all cases. Two of the parameters, a and b, clearly vary, and probably h does as well.

So we have some empirical questions: within what ranges do the empirical parameters have to be to get the “shape characteristics” similar to those empirically observed. Are there some values of the parameters where there are also empirical cases where the “shape characteristics” differ? If so, then we have a test of the model in the expected correlation between those parameter sets and predictably different “shape characteristics” of the empirical cases.

Do the winding numbers help us? Not if we take the parameters as constant for a given case: all these say there is that A goes from 0 to 1 and there are any number of unstable equilibria. But now take each of the parameters and create a triangular space in which to study the winding numbers. And so forth. It seems there is something there to learn.

Thus, I would be interested in a more serious evaluation of Turchin’s work.

Now, let’s imagine that all the winding number work, parameter bounding, perturbation and parameter estimation for the empirical cases is done, and we have a “test bed” of cases over time periods where the parameters are known (they are either constant, or we know, independently of the model, when they change, e.g., changes in frontier size). In fact, Turchin does some of the parameter bounding and perturbation tests in his chapter 4. Chapter 5 provides the following ordinal-level approximations of his variable a (p. 79-82), geopolitical pressure, for 50 cultural regions, coded over 19 centuries (Appendix B).

- a. geopolitical pressure from the hinterland codes (frontier intensity): a sum of four measures (0-9)
  1. Religious difference (0-3; pp. 79-80)
  2. Linguistic distinctions (0-2; p. 80)
  3. Economic lifeway distinctions (0-2; p. 80)
  4. Warfare intensity (0-2; p. 80), in this case
    - 0 = no frontier or closure of the frontier, stateless hinterland ( $b=0$ )
    - 1 = raiding, no frontier depopulation
    - 2 = intense warfare resulting in detectable depopulation
- b. Turchin is beginning to measure b, the width of the frontier zone, for cases where data exists. E.g., for the Roman/German frontier it is about 100-200km wide.
- h. The spatial scale variable h that translates A into geopolitical power is measurable as the transport cost with a given technology through the terrain, e.g., cost for the transport of soldiers and grain to feed them.

$r_0$ . The maximum relative growth rate of C that takes place right at the border  $x=A$ , presumably as a resistance to the opposing group along that frontier, which in the case of conflict is measured by the cohesion of the military forces encamped there.

All these are measurable variables, including  $r_0$  which seems to be partly a function of the size of the frontier. The model then is predictive of individual cases with no free parameters, and no curve-fitting, although  $a$  and  $b$  might change over shorter time scales, and  $h$  over longer time scales. The predictions for agrarian states are the test of the model. Note that we do not have to measure C because its growth and decline is also modeled as a function of the other variables. And if this predictive test is passed, then it might also be worthwhile to measure C (or my K) directly, where possible.

What are the empirical predictions, within error bounds for the measured variables, what does this tell us? Let's look at equations 4.5 again.

$$\dot{A} = \Delta A = AC(1 - A/h) - a$$

$$\Delta C = r_0(1 - A/2b)C(1 - C) \quad (4.5)$$

Territorial size A is a function of the distance to an ethnic frontier, with the default value of  $A = \infty$  where no frontier exists. A basic qualitative prediction in this case is that empires shrink or never come into existence at all if no inter-polity frontier exists. This is tested empirically (p. 84) for the first millennium CE (Pearson's  $\phi = .76$ ,  $p < .0001$ , 5 exceptions of 50) and the second millennium ( $\phi = .64$ ,  $p < .0001$ , 9 exceptions of 50). For the first millennium there were four exceptions where a region was incorporated into its neighboring empire. One, Aquitaine, is a remnant of a preexisting empire that collapsed. As for exceptions in the second millennium, four regions were incorporated into a neighboring empire, and two others that had sizeable territories and ethnic frontiers did not expand to large territories, both in rugged mountainous territories. Two of the three cases were empires developed with ethnic frontiers are genuine anomalies, and one is regarded by Turchin as a "clear indication that the frontier model cannot be taken as a universal model of how all states arise" (p. 89: Spain ceded Sicily to the Count of Savoy in 1713 as part of the Peace of Utrecht at a time when neither Sicily nor Savoy had an ethnic frontier; it was returned to Spain in 1718 and Spain passed it along to Austria in 1720 but then declared war on Austria in 1733 and recaptured Naples and Sicily in 1738 whence Charles III was crowned ruler of the Kingdom of the Two Sicilies. Following Napoleon's occupation of the Two Sicilies and the Republican uprising of 1848, Garibaldi and his followers overthrew the French on behalf of the Kingdom of Savoy in 1860 founding the Italian nation). Ninety-nine out of a hundred predictions or reconciled exceptions is not a bad outcome of a theory test, however. I would be more inclined to view the anomalous Savoy-Sicily-Italy (SFR) polity expansion, although complex, as involving ethnic frontiers, however, rather than an exception. Collin's positional advantage model is tested for  $N=50$  (p. 90) and in contrast found to have no predictiveness.

There is in addition a two-threshold model of growth: one threshold reached when  $A/h$

reaches a value where  $AC(1 - A/h) = a$ . This is an unstable equilibrium for A. Perturbations can lead to either growth or decline. The statistical prediction, then is that growth or decline in A will continue until the transition point is reached. There is no real possibility of prediction, however, of whether the unstable equilibrium will end on a growth or decline phase. That prediction, on the basis of these variables, should not be part of the statistical test.

The second threshold for C is similar: it is reached when  $A/2b = 1$ , i.e.,  $A = 1/2b$ . This is an unstable equilibrium for C. Perturbations can lead to either growth or decline in C, which will lead to a change in  $AC(1 - A/h)$  relative to a and hence a specific prediction about growth or decline in A or, possibly, disruption of an unstable equilibrium for A in a predictable direction.

We ought to be able to learn more in the winding number phase of this research about how changes in variables a, b and h would affect growth or decline in A.

The only critique I have of Don's talk is that (1) he did not clarify the use of measurable variables across a sample of cases but assumed the social scientist was fitting a model to a single case, (2) he attributed too simple a model to Turchin to give us useful specific feedback on Turchin's way of approaching historical dynamics and (3) didn't give us any principled guidelines as to when, in his mind, it is legitimate to include the variables A,  $A^2$ , etc., when studying changes in some variable A. But I would not have expected Don to take the time at this juncture to get into the details of a specific model, as I try to do here.

What Don did do, quite successfully, is give us enough guidance as to first principles in the mathematics of dynamical modeling so that we as social scientists might be able to take first steps to further advances in modeling dynamics for problems such as those studied by Turchin.

What I would like to see from Don is how to start with a two-variable space in which  $0 - \text{Small} - \text{Amax}$  is a dimension of area growth, and  $0 - F$  is existence of an interethnic frontier, where we know the local equilibria at the corners. And what can we learn from winding numbers by a 5-variable dynamical model, where a, b, h, C and A are all empirical variables.

Saari's general objection may remain of course: the model is too specific, not sufficiently general. So the question is: how to back off and get generality versus use winding numbers in the 5-variable model to explore the possibility of missing equilibrium points.

On that score: Once a region with possibly missing equilibria is identified, it is clear that the researcher should obtain more precise and more extensive data to explore that region. But might the equilibrium involve a missing or unspecified variable? Do winding numbers help us there?